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Brief article

Scale-invariance as a unifying psychological principle

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Abstract

How can the classical psychological laws be explained and unified? It is proposed here that scale-invariance is a unifying principle. Distributions of many environmental magnitudes are observed to be scale invariant; that is, the statistical structure of the world remains the same at different measurement scales [Mandelbrot, B., 1982. The Fractal Geometry of Nature (2nd Edn.). W.H. Freeman, San Francisco, CA; Bak, P., 1997. How Nature Works: The Science of Self-organized Criticality. Oxford University Press, Oxford, UK]. We hypothesise that the perceptual-motor system reflects and preserves these scale invariances. This allows derivation of several of the most widely applicable psychological laws governing perception and action across domains and species (Weber's, Stevens', Fitts' and Piéron's Laws). We suggest that these fundamental laws reflect accommodation of the perceptuo-motor system to the scale-invariant physical world and therefore have a common foundation. © 1999 Elsevier Science B.V. All rights reserved.

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The perceptuo-motor system must represent the statistical structure of the environment. The present paper addresses the relation between this environmental structure and the functioning of the perceptuo-motor system. Many aspects of the environment are statistically self-similar: that is, their structure is invariant over change of scale (Fig. 1). For example, the power spectrum of natural images is invariant over changes of scale (Field, 1987), and many natural structures are scale-invariant (Mandelbrot, 1982; Meakin, 1998). Thus the shape of a cloud may give no clue as to its size; if no other cues are available a small cloud 20 m away may be indistinguishable from a large cloud 2000 m away. Similarly, from an aeroplane it is difficult to judge one's height above the sea, but not so difficult to judge one's

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height above a city – because waves do not have a characteristic size, but houses and cars do. A signature of scale-invariant systems is a power law relationship between event frequencies and event magnitudes (Fig. 2), and such relationships are observed in many physical systems including earthquake energies (Gutenberg and Richter, 1949; Johnstone and Nava, 1985), pulsar velocity glitches (Garcia-Pelaya and Morley, 1993), and many others (Bak, 1997). Similar distributional laws hold in a wide range of physical, social and economic contexts (Zipf, 1949; Bak, 1997; Ijiri and Simon, 1977; Mandelbrot, 1982).

If psychological processes are adapted to the statistical structure of the environment (Shepard, 1987; Anderson, 1990), it is possible that perceptual and motor processing systems reflect the scale-invariance of the environment. Consistent with this, a degree of scale-invariance is evident in perceptuo-motor function. For example, ambient luminance varies by a factor of up to 10 000 in moving from sunlight to shade, but the perceived brightnesses, colours and contrasts of visual stimuli are largely unaffected because of adaptation in the perceptual system (e.g. Fechner, 1860; MacKay, 1963). In general, then, perception appears to be sensitive to ratios between stimulus magnitudes rather than absolute stimulus magnitudes – that is, the output of many aspects of perception appear be independent of absolute scale. Auditory pattern recognition is close to scale invariant with respect to frequency – in music perception, the same 'tune' is heard if the frequency of all notes is changed by the same factor. In complex movements, such as handwriting, the detailed spatial and temporal structure of actions remains invariant over substantial changes of speed and spatial scale (Viviani and Terzuolo, 1980). However, more importantly, the assumption of scale-invariance in perceptuo-motor systems allows the derivation of several classical psychological laws.

Consider the implications of scale-invariance for the accuracy of perceptual encoding (Fig. 3). Neural coding must have finite precision, so measurement error

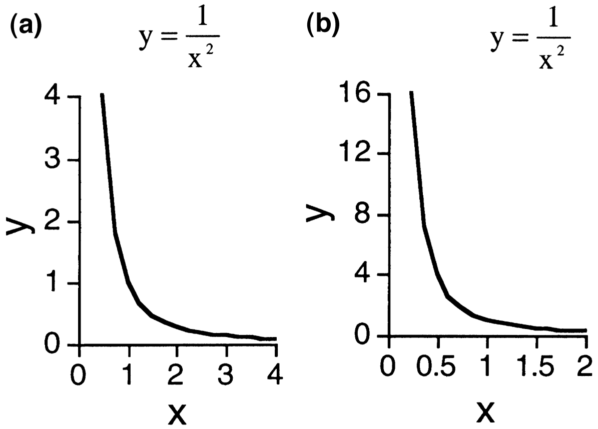

Fig. 1. The concept of scale invariance. (a) and (b) show the same function, $y = (1/x^2)$ plotted on different scales. Note that the functions are visually indistinguishable. This illustrates the meaning of 'scale-invariance' – the form of the function gives no information about the absolute value of the magnitudes involved, and hence gives no clue regarding the scale of the units involved.

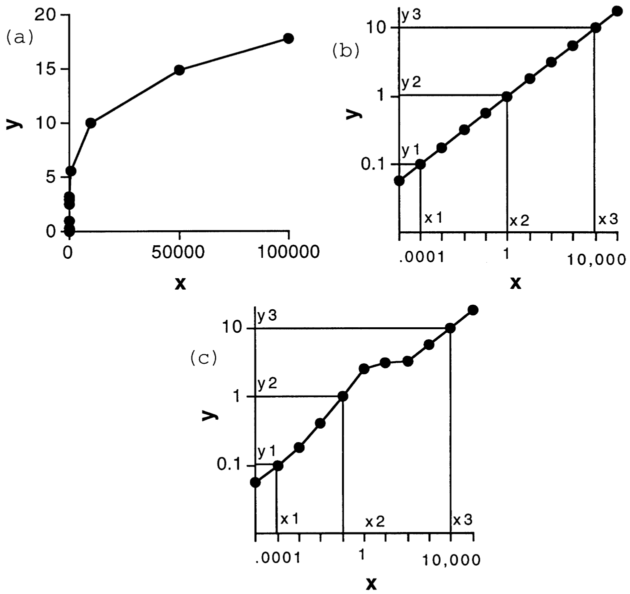

Fig. 2. Power-law relationships and scale invariance. (a) shows the power-law relationship $y = x^{0.25}$, and (b) shows the same relationship with logarithmic transformations of both axes. Note that the resulting straight line implies ratio preservation: $y_3/y_2 = y_2/y_1$, and correspondingly $x_3/x_2 = x_2/x_1$. When the original relationship is not a power-law, in contrast, then the log-log plot is no longer linear (c). Consequently ratio preservation no longer occurs: $y_3/y_2 = y_2/y_1$, as before, but now $x_3/x_2 \neq x_2/x_1$. The changing ratios in this case provide evidence regarding the absolute magnitudes of x and y.

is inevitable. Assuming that the measuring system is scale invariant, the distribution of errors cannot reveal the absolute magnitude, I. This means that the width of the error distribution cannot carry information about the absolute size of the magnitude being measured. This implies that the proportional error (i.e. standard deviation of the measurement divided by magnitude) is constant – that is, error is proportional to absolute magnitude. Suppose, by contrast, that the measuring system has 0.01 proportional error in judging, say, lengths around 1 m; 0.02 proportional error for lengths around 10 m, 0.03 accuracy for lengths around 100 m, and so on. Suppose further that a particular rod is judged several times to have a length of 0.5, 0.505, 0.495 and so on, measured using unknown units; the proportional error is estimated at 0.01. We are then able to determine from these measurements that the length being measured is about 1 m long. Thus we could determine the scale purely from the data, in violation of scale invariance. However, if scale invariance holds, proportional error cannot carry any such information, and hence proportional error must be constant.

In judging perceptual magnitudes, this simply amounts to Weber's Law, which states that measurement error, ΔI, is proportional to I. This is among the most widely cited of all psychological laws (Laming, 1986), applying to an approximation to almost all sensory dimensions. Thus the assumption that the perceptual system reflects scale-invariance in the environment predicts the default relationship govern-

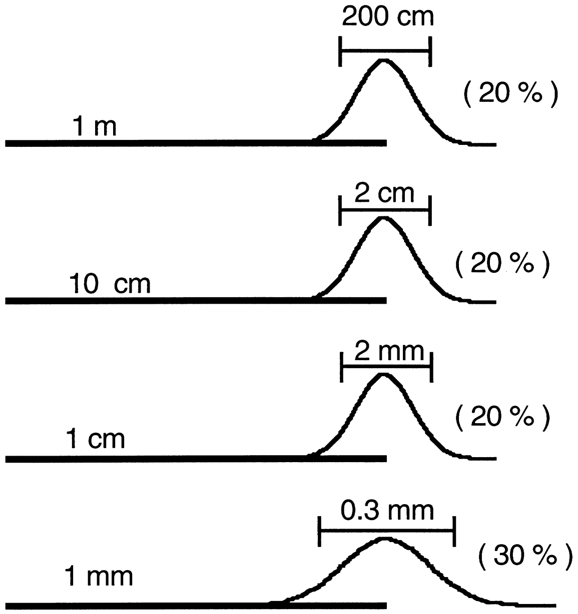

Fig. 3. Measurement error and scale invariance. The graph illustrates how scale invariance implies Weber's Law. A hypothetical measuring instrument is used to measure various lengths repeatedly. For the same length, the instrument gives numerical outputs which vary from trial to trial, due to measurement error. If the instrument's behaviour is scale-invariant, then it should not be possible to tell the scale of these numerical outputs. This means that the percentage error at different scales must be identical – indeed, the error distribution should have exactly the same shape (as shown for the top three distributions). But if the percentage error (e.g. for very small objects) changes (here, from 20% to 30%) then this gives information about the scale at which the instrument is working (e.g. it is measuring at the order of mms). Such violations of scale-invariance are inevitable at the extremes of the range of any measuring instrument, including the perceptual system.

ing measurement error and magnitude in perception (Weber's Law). For any measuring system, of course, scale-invariance will be violated as the limits on discrimination imposed by the physical structure of the sensory organs are approached. Thus proportional error increases markedly when stimuli are barely detectable. Similar breakdowns of Weber's Law are typically observed for stimuli at very high magnitudes, beyond the normal operating range of the sensory system. Thus, we would expect a plot of Weber fraction against absolute magnitude to be a composite function, consisting of a linear relationship in the normal working range of the perceptual system, where Weber's Law holds, with concave-upward 'tails' at the extreme ends of the magnitude range, where proportional error is raised (this pattern is shown in Fig. 4). It has been argued (see e.g. Krueger, 1989) that the appropriate underlying function is not 'linear plus tails', but some other function (e.g. one in which error is proportional to the intensity I raised to some power: Guilford (1932); or added to some constant: Piéron (1952)). There is considerable debate concerning which account is most appropriate (Krueger, 1989; Laming,

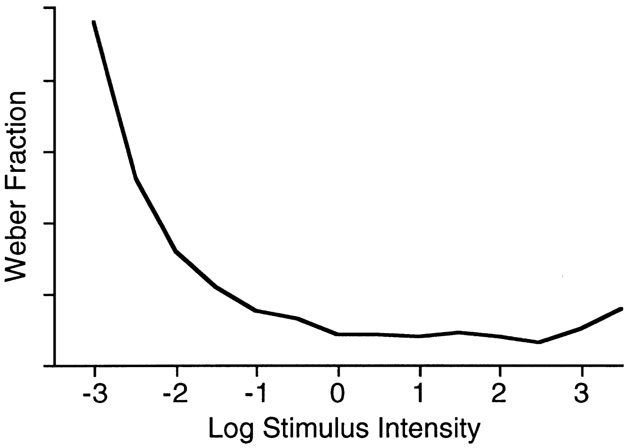

Fig. 4. Departures from Weber's Law. Hypothetical function relating sensitivity, $\Delta I/I$, to log stimulus intensity, I. Discriminability is less for very small or large stimulus magnitudes (see text for details), but is approximately constant (conforms to Weber's Law) over several orders of magnitude throughout the normal operating range. The schematic curve is based on sensitivity to white light (see e.g. Geldard, 1972).

1997). The predictions derived from the scale-invariant character of the environment as derived here are confirmed only to the extent that Weber's Law is, within the normal operating range of the sensory system, true. However we also note the difficulty of excluding the possibility that minor deviations from Weber's Law within the normal operating range may reflect the constraints imposed on adapting systems by biological/implementational considerations.

More generally, scale invariance also implies that the probability of correctly distinguishing the larger of a pair of presented magnitudes should depend only on the relative difference between the magnitudes. This prediction is a much stronger test of scale invariance in perception, because it requires the invariance of an entire probability distribution, rather than the single quantity of proportional error. The discriminability function (the probability that $I + \Delta I$ is categorized as having greater magnitude than I, for varying ΔI) does indeed have an invariant form (typically a cumulative normal) whatever the value of I, at least in those areas where it has been investigated (e.g. for discrimination of sound pressure: Green and Sewall (1962); Green (1967); and for intensity of odour: Stone and Bosley (1965); see Laming (1986) for discussion).

Finally, we note that an analog of Weber's Law is observed in motor control: duration and force in ballistic movements are known to have a variance proportional to magnitude, and accuracy is proportional to distance in a range of aiming tasks (Schmidt et al., 1979).

Scale independence also has implications for tasks in which people match perceptual or motor magnitudes to numerical values or to each other. If the mapping between two real-valued dimensions, X and Y, is scale invariant the mapping must be ratio-preserving (Fig. 2). If this were not the case, then information about absolute scale could be obtained by observing how these ratios varied. To see why, let us assume that X and Y, measured in units u and v respectively, are related by an exponential relationship $Y = 2^X$, where ratios are not preserved. A constant ratio change in X (e.g. 1:2) corresponds to very different ratio changes in Y, for different absolute magnitudes (thus contravening scale invariance). Thus, if X changes from 1 u to 2 u (a 1:2 ratio), Y changes from 2 v to 4 v (a 1:2 ratio); but if X changes from 2 u to 4 u (still a 1:2 ratio), Y changes from 4 v to 16 v (now a 1:4 ratio); and if X changes from 4 u to 8 u (again a 1:2 ratio), Y changes from 16 v to 256 v (now a 1:16 ratio). This means that ratio changes carry information about the absolute magnitudes being measured. So, for example, if we are given magnitudes of 0.5 and 1 in X (but units are not known), and these correspond, respectively, to 500 and 2000 in Y (units not known), we can determine the absolute magnitudes, by noting that a 1:2 ratio in X corresponds to a 1:4 ratio in Y, which indicates that the values X must be about 1 v and 2 v, and the Y values must be about 4 v and 16 v. Thus we have determined absolute magnitude (and can trivially determine the measuring units) from the relationships between these ratios. This is exactly what scale-invariance rules out.

If the perceptual system is scale invariant, and there is no information in the ratios between values of X and Y, then X and Y must stand in a power law relation: $Y \propto X^{\mathrm{k}}$, where k is a constant. If this is true, then if two values of X differ by a ratio r, then the two corresponding values of Y differ by a ratio of r^{k}, independent of the absolute values of X and Y. Therefore the ratios between pairs of X and Y values carry no information about the absolute magnitudes of X and Y when a power law relation holds between them.

Thus, scale invariance predicts that a power law relationship should hold in matching different perceptual or motor magnitudes, and in matching these to numerical values. This is Stevens' Law (Stevens, 1975), which has been empirically confirmed for over 30 perceptual and motor dimensions[1].

Similar arguments apply to the case of motor control. Perceptual-motor activity takes place in time, and the temporal behaviour of organisms is often scale-invariant (Gibbon, 1977). The assumption of temporal as well as spatial scale independence has implications for the relation between time-accuracy relations in perceptuo-motor control. Consider an effector being moved a distance d to a target of size w. To preserve spatial scale invariance, the time to make this movement must be a function of d/w, because otherwise changes in movement times would not be independent of length measurement scale. If movement is also scale-invariant in time (Viviani and Terzuolo, 1980), then movement time should be a power law function of d/w. This model fits closely with over 40 diverse experiments on perceptuo-motor control, ranging from dart-throwing to control in microscopy – it is one version of Fitts' Law (Fitts, 1954). Harris and Wolpert (1998) show that the velocity profiles of eye and arm movements, and Fitts' Law, can be explained on the assumptions that neural control signals are subject to noise that increases in proportion to signal magnitude and that the motor control system minimises the resulting final position

[1]Note that this explanation does not assume the existence of an internal sensory scale (Laming, 1997).

error. Thus scale-invariance at the neural level is also consistent with the observed functioning of the perceptuo-motor system.

Finally, consider expected reaction times, $E(T)$, in detecting perceptual stimuli with a magnitude (e.g. luminance, sound pressure), I. Large I are detected more rapidly than are small I. A crude application of scale invariance implies that $E(T)$ is a power law function of I. But RT does not tend to zero however large the stimulus to be detected, because the perceptuo-motor system takes a certain time to initiate any response. Therefore a constant response time, r_0 must be subtracted from each reaction time, T. The residual time should, by scale-invariance, be a power function of the magnitude to be detected. This gives Piéron's Law, $E(T) - r_0 = \mathrm{k}I^{-\beta}$, where k and β are constants (Piéron, 1952).

Clearly, the perceptuo-motor system is not completely scale invariant – for example absolute magnitude judgements are possible, albeit remarkably poor (Garner, 1962), and invariance inevitably breaks down at extremes of perceptuo-motor function. Nonetheless, many aspects of perceptuo-motor function appear to reflect the scale-invariant character of the physical world to a remarkable degree, suggesting that scale-invariance may be a unifying psychological principle that underpins some of the most fundamental psychological laws. Finally, the fact that scale-invariance breaks down radically in some cases (e.g. that of colour vision) argues against the possibility that the scale-invariance that is normally observed simply reflects the operation of physical systems which, being subject to the laws of nature, must themselves embody scale-invariance.

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