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Computational Levels and Conditional Inference: Reply to Schroyens and Schaeken (2003)

Mike Oaksford
Cardiff University

Nick Chater
University of Warwick

This article is a reply to W. Schroyens and W. Schaeken's (2003) critique of M. Oaksford, N. Chater, and J. Larkin's (2000) conditional probability model (CP) of conditional inference. It is argued that their meta-analysis does not falsify CP because the evidence may bear on more than one computational level of explanation. Moreover, it is argued that CP provides a rational account of more of the data than W. Schroyens and W. Schaeken's mental models theory. Other points are also addressed. It is suggested that W. Schroyens and W. Schaeken's model and CP converge on the importance of probabilistic prior knowledge in conditional inference. This is consistent with the normative literature, which (like CP) treats conditionals in terms of subjective conditional probabilities.

Oaksford, Chater, and Larkin (2000) presented a model of human reasoning with conditionals (if *p*, then *q*) based on subjective conditional probability, $P(q|p)$, which accords with the normative philosophical literature (Adams, 1966, 1975; Edgington, 1995), where "the majority view [is] that straight [i.e., indicative] conditionals are a matter of subjective conditional probabilities" (Bennett, 1995, p. 332).[1] However, good philosophy need not necessarily translate into empirically sound psychology. Indeed, Schroyens and Schaeken (2003) argued that our conditional probability model (CP) is not consistent with their meta-analysis (Schroyens, Schaeken, & d'Ydewalle, 2001), and that an alternative, validating search model (VS) that supplements mental models theory (MM; Johnson-Laird & Byrne, 1991) provides better fits to the data. Before dealing with each specific point that Schroyens and Schaeken raised, we consider how their two main points should be interpreted in view of the standard scheme for computational explanation in cognitive science (Anderson, 1990; Marr, 1982).

Computational Explanation and Rationality

The conditional inference task involves presenting participants with a conditional premise (if *p*, then *q*), a categorical premise (one of *p*, not-*p*; *q*, or not-*q*), and a conclusion (*q*, not-*q*; *p*, not-*p*). There are two logically valid inferences: modus ponens (MP; if *p*, then *q*, *p*, therefore *q*) and modus tollens (MT; if *p*, then *q*, not-*q*, therefore not-*p*). The remaining inferences are logical fallacies, denying the antecedent (DA; if *p*, then *q*, not-*p*, therefore not-*q*) and affirming the consequent (AC; if *p*, then *q*, *q*, therefore *p*). The rules can also be presented with negated constituents to produce three more rules: If *p*, then not-*q*, if not-*p*, then *q*, and if not-*p*, then not-*q*. These rules lead to corresponding changes in whether the categorical premise and conclusion of each inference is negated or not.

Schroyens and Schaeken indicated that their meta-analysis of conditional inference with negations (Schroyens, Schaeken, & d'Ydewalle, 2001) confirmed five of the six predictions made by CP. Moreover, they showed that VS provided better fits to the purely affirmative data than CP. We examine these two points in order.

Failed Predictions

We make two observations. First, for a straightforward application of a normative theory in psychology, five of six confirmed predictions is rather good performance. For example, the normative theory on which mental models is based (i.e., standard logic and material implication[2]) accounts for as little as 4% of people's behavior in the other main conditional reasoning paradigm, Wa-

Mike Oaksford, School of Psychology, Cardiff University, Cardiff, United Kingdom; Nick Chater, Department of Psychology, University of Warwick, Coventry, United Kingdom.

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Correspondence concerning this article should be addressed to Mike Oaksford, School of Psychology, Cardiff University, P.O. Box 901, Cardiff, CF10 3YG, Wales, United Kingdom. E-mail: oaksford@cardiff.ac.uk

[1] All the conditionals used in these experiments are "straight" or "indicative" conditionals. We parameterized our model not in terms of $P(q|p)$ but $P(\text{not-}q|p)$, that is, $1 - P(q|p)$. In the normative literature, this is termed the *conditional uncertainty* (Edgington, 1995). In our model we referred to it as the "exceptions parameter."

[2] According to this theory, a conditional (if *p*, then *q*) is true if *p* is false or *q* is true. That is, mental models theory unquestioningly assumes that a truth functional approach can be adopted to all conditionals, including those that occur in everyday life. However, if these everyday conditionals must be interpreted in terms of conditional probability, as assumed in the normative literature, one consequence is that people "do not use 'if' to express propositions, evaluable in terms of truth" (Edgington, 1995, p. 280). This means that the mental models approach is at odds with current thinking about conditionals in the philosophy of language and logic.

son's (1966, 1968) selection task.[3] Moreover, standard logic fails to make any of these six predictions (see *MT Referred Clause Effect* section).

Second, falsifying data can miss its target because it bears not on the theory but on some auxiliary assumption (Lakatos, 1970; Putnam, 1974). For a cognitive psychologist, cognition is computation (Oaksford & Chater, 1991; Pylyshyn, 1984). Computational explanation is multileveled (e.g., Anderson, 1990; Marr, 1982; Pylyshyn, 1984). The computational (Marr, 1982) or rational (Anderson, 1990) level specifies a normatively justified and descriptively adequate model of some cognitive phenomenon (Oaksford & Chater, 1996, 1998). That is, together with some assumptions about the nature of the environment (see *The Rarity Assumption* section), it can be shown that behavior approximates optimal performance on the task. Such an account should provide a rational understanding of most of the data. The computational level theory is implemented at the algorithmic or performance level. Oaksford et al. (2000) presented an account of conditional reasoning in which the computational level theory is provided by Bayesian probability theory. Consequently, Schroyens and Schaeken's single predictive failure only hits its target on the auxiliary assumption that the algorithmic level has no work to do. Oaksford et al. (2000) carefully avoided this assumption. It is only by assuming that Oaksford et al. (2000) must explain all the results at the computational level that Schroyens and Schaeken's results falsify CP. This may seem unreasonable when it is considered that mental models theory is an algorithmic level theory that invokes various processing assumptions to compensate for the descriptive inadequacy of material implication (see footnote 2). Consequently, the one explanatory failure Schroyens and Schaeken found is best interpreted as indicating that we need to provide an algorithmic level theory of conditional inference; that is, our theory "has to allow for a processing account" (Schroyens & Schaeken, 2003, p. 148). This is a view with which we wholeheartedly agree. However, it does not mean that CP is false, as Schroyens and Schaeken claim. Moreover, the same line of argument would imply that the material implication interpretation of the conditional, which fails to make any of these six predictions, is false and hence, a fortiori, that mental models theory, which assumes this interpretation, is false also.

Model Comparison

The issue of computational levels of explanation also bears on how to interpret Schroyens and Schaeken's model comparison exercise. They show that VS, an algorithmic level theory, accounts for 92.7% of the variance, whereas CP, a computational level theory, accounts for 84.5%, a modest (8.2%), albeit highly significant, improvement. However, this improvement comes at a cost. We argue that there are three hierarchically organized possibilities. First, despite appearances, holding to the mental models account actually represents a reduction in the amount of people's behavior that can be rationally explained. Second, the first possibility can be rejected but only at the cost of attributing people with inconsistent beliefs. Third, the first and second possibilities can be rejected but only at the cost of abandoning the multileveled scheme of computational explanation assumed in the cognitive sciences. None of these alternatives seem attractive. We now show how these consequences arise.

The computational level theory of MM is the truth functional account given by standard logic (see footnote 2). What proportion of the variance can be explained by this theory? According to standard logic, the rule should be interpreted as material implication: Endorse only MP and MT. However, it has been argued that many people interpret the conditional if *p*, then *q* as implicating if *q*, then *p*. This interpretation is called a *biconditional*. However, it is nonnormative to assume this additional possible interpretation because if *p*, then *q* does not logically entail if *q*, then *p*. Nonetheless, we will allow the predictions of the biconditional interpretation (i.e., endorse all inferences) to count as normative for the purposes of this comparison. Consequently, MP and MT should always be endorsed, but DA and AC will be endorsed only on the biconditional interpretation. Allowing for errors means that MP and MT may not always be endorsed. Let the proportion of valid inferences be P_V and the proportion of people adopting the biconditional interpretation be P_B.[4] The least means squares fits for this logical model are given by the mean of MP and MT for P_V and the mean of AC and DA for P_F. R^2 was then calculated for each rule in Schroyens and Schaeken's meta-analysis. The mean R^2 was .56 (SD = .31), which is significantly lower than the mean value for the probability model of .85 (SD = .14), $t(64) = 6.73$, $p < .0001$. This means that by assuming CP a significantly greater proportion of the variance in these data (an extra 29%) can be understood as rational behavior (i.e., can be explained by a normative theory).

Perhaps, however, Schroyens and Schaeken intend VS to be normatively justified but not by standard logic. Indeed, in VS people check world knowledge for counterexamples for MP and MT. However, if they believe the premises to be true and interpret the conditional as material implication, then these counterexamples cannot exist. Consequently, Schroyens and Schaeken must believe that the extra variance VS accounts for is explained by a different normative theory. Indeed, it seems that they regard VS as justified by probability theory. This is consonant with their argument that theories of reasoning must address the probabilistic component of human reasoning and that CP and VS are very

[3] Oaksford and Chater (in press-b) fit a processing tree (Batchelder & Riefer, 1999) implementation of mental models theory to their meta-analysis of selection task data (Oaksford & Chater, 1994) and show that it could be rejected. In contrast, their optimal data selection model (Oaksford & Chater, 1994), which uses the same conditional probability interpretation of the conditional as CP, could not be rejected. The mental models account did not include a validating search process but just the standard mental models framework for constructing initial representations of the conditional or biconditional, which may or may not be fleshed out. Formally, except for the MT inference, this model was identical to VS (Oaksford & Chater, in press-a). That is, the same equations are used, although the parameters have different interpretations. This model can be directly applied to the conditional inference task where it does almost as good a job as VS (Oaksford & Chater, in press-a). This seems to mean that either the standard mental models account or VS is explanatorily redundant; either can explain the data on the affirmative, abstract conditional inference task.

[4] Errors can also occur for DA and AC. However, on the conditional interpretation, an error means drawing DA and AC, whereas on the biconditional interpretation, an error means not drawing DA and AC. Hence, for simplicity we assumed that errors cancel for DA and AC.

similar (Schroyens & Schaeken, 2003).[5] Remaining as consistent with MM as possible, the parameters of VS could be interpreted rationally in terms of the appropriate subjective probabilities specified by material implication. According to such an account, the probability of a conditional is 1 minus the probability that there is a true-antecedent and false-consequent case (see Evans, Handley, & Over, in press). The parameters of VS then correspond to the probabilities of finding a *p*, not-*q* instance (E_{TF}), a not-*p*, *q* instance (E_{FT}), and a not-*p*, not-*q* instance (W_{FF}). If these probabilities are rationally constrained by the probabilities specified by material implication, then $E_{TF} = P(p, \text{not-}q)$, $E_{FT} = P(\text{not-}p, q)$, and $W_{FF} = P(\text{not-}p, \text{not-}q)$. However, according to Schroyens and Schaeken's model fits, $E_{TF} = P(p, \text{not-}q) = .048$, $E_{FT} = P(\text{not-}p, q) = .348$, and $W_{FF} = P(\text{not-}p, \text{not-}q) = .778$. To be a consistent probability model, $P(p, q) + P(p, \text{not-}q) + P(\text{not-}p, q) + P(\text{not-}p, \text{not-}q) = 1$. However, this means that according to Schroyens and Schaeken's model fits $P(p, q) < 0$, which violates the axioms of probability theory. Thus, providing a rational basis for VS has the paradoxical consequence that people must be attributed with inconsistent beliefs.[6]

Another possibility, however, is that Schroyens and Schaeken can dispense with a rational justification for their model. This is consistent with their dismissal of the rationality issue and their argument that the main goal of the psychology of reasoning is "the development of descriptively adequate cognitive processing systems" (Schroyens & Schaeken, 2003, pp. 147). However, this would mean abandoning the multileveled explanatory scheme adopted in the cognitive sciences, which was a reaction against such unprincipled processing theories in the past (Anderson, 1990; Marr, 1982). Without a normatively justified computational level theory, there is no explanation of why people's reasoning is generally successful (Chater, Oaksford, Nakisa & Redington, in press). Moreover, it is misleading to argue that the psychology of reasoning is not concerned with rationality. Mental models theory is committed to the standard truth functional view of the conditional. This is perhaps bought into starkest relief by mental models research on illusory inferences (for a brief summary, see Johnson-Laird, 2001). These are inferences that are logically invalid but which people are inclined to make apparently because of their mental model representations. This behavior can only be viewed as "illusory" if the researcher is working with some conception of the inferences people should make. It is not an option to say that this is irrelevant! This is because, from the standpoint of a different normative theory, "illusions" may be valid inferences. However, as we have seen, VS appears to reject the standard truth functional account. Thus, Schroyens and Schaeken seem to be on the horns of a dilemma. They could reject normative theories, in which case they cannot explain why these inferences are illusions, or they could propose a different normative theory, which means it is possible that these are not illusions at all.

To summarize, the improved fit for VS comes at a cost. Either the theory is not rationally justified, because Schroyens and Schaeken abandon the standard account of computational explanation in cognitive science, or it is rationally justified but only at the cost of attributing people with inconsistent beliefs. We suggest that these arguments show that CP should be preferred as the computational level explanation of these data because it provides a rational explanation of more of the results. However, as Oaksford et al. (2000) conceded, an algorithmic level implementation is required. We now address the other points raised by Schroyens and Schaeken in detail.

Further Issues

Schroyens and Schaeken raise a number of further issues in the *Conditional Reasoning With Negatives* section and the *General Discussion* sections. We discuss these now.

Conditional Reasoning With Negatives

MP inference rates. Schroyens and Schaeken's meta-analysis revealed a high acceptance rate for MP (94.3%). This seems incompatible with CP because it suggests that $P(q|p)$ is high and so the probability of exceptions ($P(\text{not-}q|p)$) is low. However, to explain why MT inferences are made less frequently than MP, CP seems to require higher values of $P(\text{not-}q|p)$. We agree that CP underestimates MP endorsements. However, consistent with Schroyens, Schaeken, Fias, and d'Ydewalle's (2000, p. 1729) claim "that even within the context of abstract reasoning problems, people call on their knowledge and beliefs in satisfying task demands" there could be another feature of people's prior knowledge that explains why they ignore exceptions more for MP than for MT.

Take Rips's (1994) example: "If Calvin deposits 50 cents, he gets a Coke." Suppose Calvin believes that this Coke machine is faulty, that is, $P(\text{Coke}|\text{Calvin deposits 50 cents}) = .75$. Nonetheless, the only way he can find out whether he is going to get a Coke this time is to deposit his 50 cents. That is, even with rules known to admit exceptions, people often have to act in daily life just as they would if they believed the rule to be exceptionless. In contrast to this forward style of reasoning, Calvin may reflectively consider the causes of why his colleague is not in possession of a Coke. If she actually wants a Coke, then it is reasonable to assume that she has tried depositing her 50 cents. That is, Calvin might be unlikely to conclude that the explanation for his colleague not possessing a Coke is that she failed to deposit her 50 cents. He is more likely to

[5] Schroyens and Schaeken do not present their model fully. However, they do refer the reader to Schroyens, Schaeken, and d'Ydewalle (2001b), in which a 14-parameter processing tree model (Batchelder & Riefer, 1999) is presented. Once a penalty is introduced for the number of free parameters, CP would win a competition with this model with ease (although there are insufficient data points, i.e., 4, to constrain a 14-parameter model). Schroyens and Schaeken argue that the three-parameter model they present is derived from this model. However, they do not present it as a processing tree model, so it is not clear precisely how the equations for the probability of drawing an inference are arrived at. Even if they correspond to the arcs of a processing tree model, these only indicate how likely a particular process is to be enacted, and so such a model is a rational model only if the processes themselves are rationally justified.

[6] If VS is made to conform to the probabilities specified by the material conditional, then the result is the logical model specified previously. However, the equations of VS could be retained but with the constraint that $E_{TF} + E_{FT} + W_{FF} < 1$ imposed. We fitted VS imposing the constraint that $E_{TF} + E_{FT} + W_{FF} \leq (N - 1)/N$, where N is the sample size for the study, so $1/N$ is the lowest value $P(p, q)$ can reasonably take. The mean R^2 was .78 (.16), which was significantly lower than for CP, $t(64) = 2.88$, $p < .005$. Thus, when the parameters of VS are rationally constrained to avoid inconsistent beliefs, it provides worse fits to the data than CP.

conclude that this time the fault prevented Coke delivery. In sum, in using conditionals in everyday life, people frequently have to act as though they were exceptionless in forward predictive reasoning and only consider the exceptions in reflective explanatory reasoning.

MT referred clause effect. Schroyens and Schaeken indicate that CP predicts a negative referred clause effect for the MT inference. However, their meta-analysis of the abstract task failed to reject the null hypothesis of no negative referred clause effect. Schroyens and Schaeken argue that this single predictive failure means that "[Oaksford et al.'s (2000)] model is false and needs to be amended if not rejected" (p. 143). However, to properly interpret this claim, CP needs to be compared with the alternative computational level theory provided by mental models. According to standard logic, people's inferential behavior should be unaffected by the presence of negations. In standard logic, *p* and not-*p*, which are called "literals" (Quine, 1950), have exactly the same logical status: That is, standard logic cannot account for any of the negations effects observed in Schroyens and Schaeken's meta-analysis. However, Schroyens and Schaeken do not then explicitly advocate abandoning standard logic as a computational level theory of these data, although, as we have seen, this seems to be a consequence of the VS model. However, in the absence of an alternative normative theory, the current explanation appears to account for all these results by the processes that implement standard logic in the mind. It is inconsistent not to allow CP the same leeway.

MT and AC inferential-negation and DA referential-negation effects. Schroyens and Schaeken observe that the inferential-negation effect was greater for MT than for AC, and that there was a significant referential-negation effect for DA. They argue that to model this pattern requires inconsistent parameter settings: To model the first effect requires that $P(q) + P(\text{not-}q|p)$ is greater than 1, whereas the second effect requires that this sum is less than 1. However, these constraints only apply assuming no between-rule variation other than that dictated by the presence of negations. Thus, for example, $P(q)$ is exactly the same for the affirmative consequent rules (if *p*, then *q* and if not-*p*, then *q*) and increases by the same fixed amount for the negated consequent rules (if *p*, then not-*q* and if not-*p*, then not-*q*). This may seem unreasonable when it is considered that Schroyens and Schaeken fit VS and CP to the data on a rule-by-rule basis (i.e., the parameters of each model are allowed to vary for each rule).

Table 1 shows the results of fitting CP rule by rule to the aggregate data that Schroyens and Schaeken report in their Figure 4. Table 1 shows the best fit parameter values and the predicted proportions of inferences endorsed. The fit was good, with an R^2 of .93 (i.e., CP captures the trend in the aggregate data well). Moreover, the root mean square deviation was .053 (i.e., on average the predicted values deviated from the observed values by about 5%). For each rule, the model must conform to the constraints Schroyens and Schaeken outline. However, the predicted MT inferential-negations effect is greater (on average 24% fewer endorsements for a negated inferential clause) than the predicted AC inferential-negation effect (on average 17% more endorsements for a negated inferential clause). Moreover, there is a small predicted referential-negation effect for DA (on average 5% more endorsements for a negated referential clause). That is, the model adequately captures the relative sizes of these effects as long as there is some between-rule variation in the parameter values.

Should such between-rule variation be allowed? All we can say is that Schroyens and Schaeken's model also has to allow such variation to provide comparable fits to the data. The results of

Table 1
The Predicted Probabilities With Which Each Inference Was Drawn for Each Rule in Schroyens and Schaeken's (2003) Meta-Analysis of the Negations Paradigm for the CP Model and the VS Model, Showing the Best Fit CP Values of P(p), P(q), and P(not-q|p) and VS values of E_{TF}, E_{FT}, and W_{FF}

	Inference				Parameters		
Model and rule	MP	DA	AC	MT	$P(p)$	$P(q)$	$P(\text{not-}q\|p)$
CP							
AA	.888	.545	.721	.786	.570	.702	.112
AN	.909	.295	.552	.772	.489	.805	.091
NA	.855	.557	.868	.528	.774	.762	.145
NN	.845	.390	.743	.546	.676	.769	.155
					E_{TF}	E_{FT}	W_{FF}
VS							
AA	.958	.553	.701	.756	.042	.299	.789
AN	.975	.367	.497	.721	.025	.503	.739
NA	.888	.524	.833	.559	.112	.167	.629
NN	.915	.404	.703	.526	.085	.297	.575

Note. Although we have retained the standard labeling for CP's parameters, because of the negations in the rules, strictly $P(p)$ corresponds to the probability of a true-antecedent case, *P*(TA). Similarly, $P(p)$ is the probability of a true-consequent case, *P*(TC), and $P(\text{not-}q|p)$ is the probability of a false-consequent case given a true antecedent case, *P*(FC|TA). CP = conditional probability model; VS = validating search model; AA = if *p*, then *q* rule; AN = if *p*, then not-*q* rule; NA = if not-*p*, then *q* rule; NN = if not-*p*, then not-*q* rule; MP = modus ponens; DA = denying the antecedent; AC = affirming the consequent; MT = modus tollens.

fitting VS rule by rule to the aggregate data on the negations paradigm are also shown in Table 1. We compared the between-rule variance in the parameter values between models. There was only one significant difference: $P(\text{not-}q|p)$ varied significantly less than E_{FT}, $F(3, 3) = .045$, $p = .030$ (the variance for the VS model's parameters was used as the denominator in the F ratio). Table 1 also reveals an interesting effect for the VS model: It apparently fails to predict the referential-negation effect for the DA inference. According to the predicted values, there are only .8% more endorsements predicted for a negated referential clause for the DA inference. That is, it seems that VS may not capture the pattern of results in Schroyens and Schaeken's meta-analysis as well as CP, even though the parameters of VS have to vary as much between rules as the parameters of CP.

Algorithmic level. Schroyens and Schaeken argue that Oaksford et al.'s (2000) proposal that at the algorithmic level people might differentially weight information about the referential and inferential clauses is inadequate. In Oaksford et al. (2000), this seemed most consistent with the irregular referential clause effects found when probabilities were manipulated. We agree that this differential weighting hypothesis could only explain the MT findings by failing to explain the results for the AC inference. This suggests that an adequate algorithmic level model needs to be provided, a point made by Oaksford et al. (2000) and one that we endorse again here.

General Discussion

Rarity assumption. Schroyens and Schaeken point out that the best fit parameter values for our probability model are generally quite high (about .5). This finding seems to violate the rarity assumption (Oaksford & Chater, 1994, 1996, 1998) that the probabilities of p and q are generally small, which we invoked to explain the pattern of performance observed in Wason's selection task (Oaksford & Chater, 1994, 1996, 1998, in press-b). As we explained in our original article (Oaksford et al., 2000, p. 897), low probabilities are overestimated in the inference task because "inferences are only relevant when the properties or events to which they apply are more likely than normal to occur." For example, suppose one knows that swans are aggressive. It seems unlikely that this information will be available unless someone is in a context in which he or she is more likely than normal to encounter swans; that is, $P(x \text{ is a swan})$ is higher than its default rarity value. Although this is true for inference, Wason's selection task engages inductive reasoning. The goal is to establish generalities (e.g., swans are aggressive) that are generally true across contexts. Across contexts, swans and aggressive things are rare. In sum, although it is rational to take account of rarity in selecting data to test hypotheses, it is usually only in contexts in which the antecedent is more likely to be satisfied than normal that people need to use the rule to draw inferences.

Implicit negations. Schroyens and Schaeken argue that Oaksford et al.'s (2000) model of the implicit negation effect appears to predict that the following two AC inferences should be endorsed equally often: If *A*, then not-2, 7, therefore *A* (implicitly negated categorical premise) and if *A*, then 7, 7, therefore *A*. This is because both inferences should be drawn in proportion to $P(A|7)$. However, in Schroyens, Verschueren, Schaeken, and d'Ydewalle's (2000) Experiment 1, the AC inference for the implicit, if *A*, then not-2 rule was endorsed by 16%, 28.6%, and 7.4% of participants across three conditions. However, the same inference for the explicit, "if *A*, then 7" rule was endorsed by 76%, 95.2%, and 70.4% of participants, respectively. A similar pattern was observed in Evans and Handley's (1999) Experiment 3 (6% vs. 89%, respectively).

The force of these results depends on the fact that the very same lexical content is used as the categorical premise and conclusion, so that each inference seems to depend on the very same conditional probability. However, in these experiments, either (a) the lexical content is randomly varied or (b) just two letters and two numbers are used with the same letter/number in the antecedent and consequent of each rule. In the latter case, the actual inferences participants would see are as follows: If *A*, then not-2, 7, therefore *A* and if *A*, then 2, 2, therefore *A*. That is, a different conditional probability is being assessed in each case. Why this matters can be seen when we substitute real-world content for the alphanumeric stimuli used in these experiments. First, we look at the two inferences that Schroyens and Schaeken present:

> *Example 1:* If you are in Paris, then you are in France. You are in France. Therefore, you are in Paris.

> *Example 2:* If you are in Paris, then you are not in England. You are in France. Therefore, you are in Paris.

Here, as Schroyens and Schaeken suggest, the conditional probability, $P(p|q)$, is exactly the same in Examples 1 and 2. Intuitively, this seems fine for CP because either example seems to provide as much warrant for the conclusion as the other. However, as we said, the lexical content normally varies. Example 2 would be as follows:

> *Example 2a:* If you are in Paris, then you are not in France. You are in England. Therefore, you are in Paris.

Now the conditional premises are contraries; that is, they violate the logical law of conditional excluded middle: not-((if p, then q) and (if p, then not-q)). Note also that although in Example 1 $P(p|q)$ will take on some positive value (most visitors to France go to Paris), in Example 2a, $P(p|q) = 0$. That is, in Example 2a, where the implicit negation is used, according to CP people should not draw this AC inference, which is exactly the result Schroyens and Schaeken describe. We now turn to what happens when the rules are not contraries, as can happen when, as discussed previously, the lexical content is randomly varied.

> *Example 2b:* If you are in Berlin, then you are not in France. You are in England. Therefore, you are in Berlin.

In Example 2b whether $P(p|q) = 0$ or has some positive value depends on which contrast class member for not-France is used as the implicitly negated categorical premise. If it is England, then $P(p|q) = 0$; if it is Germany, then $P(p|q) > 0$. These examples show that, for the cases that participants normally see, CP predicts exactly the behavior noted by Schroyens and Schaeken as a consequence of using implicitly negated categorical premises. In Example 2a one should not draw the AC inference, and in Example 2b only one member of the contrast class for the consequent will allow the inference to be drawn; the rest do not.[7]

These examples are deterministic, that is, $P(q|p) = 1$ (or 0 for Example 2a). However, we have tried substituting in different nondeterministic contents, such as causes, and the same conclusions hold up. Moreover, one effect of implicit negations may be to call to mind alternative antecedents. There are many ways of not being in France. For example, if you are in Dublin, then you are not in France. Being provided with such an alternative in the categorical premise (i.e., being in England) may bring these to mind. This may connect the effects of implicit negations to the effects of additional antecedents, which just so happen to suppress AC and DA inferences. We have addressed these effects elsewhere using CP (Oaksford & Chater, in press-c). However, this is not the place to speculate further. The point is that CP is only constrained to make the counterevidential predictions suggested by Schroyens and Schaeken when the lexical content is exactly the same in the categorical premise and conclusion of these two inferences (Examples 1 and 2). However, this is rarely the case in the actual experiments and, given Examples 1 and 2, it appears that the data in this particular case violate strong intuitions that participants should draw these inferences in equal proportion. Once the lexical content varies, as it normally does in these experiments (Examples 2a and 2b), intuition and CP converge on the observed response pattern.

Conclusion

We argue that, when interpreted at the correct level of computational explanation, Schroyens and Schaeken's meta-analysis does not falsify a probabilistic explanation of these data. Moreover, we have shown that, when compared at the same computational level, CP provides a rational explanation for more of the data than does the logical interpretation of conditionals that underlies mental models. We also showed that the attempt to provide VS with a rational basis means that people must be attributed with inconsistent probabilistic beliefs.

However, there are many areas of agreement. We are all committed to the importance of prior knowledge in human reasoning. Indeed, this has been the basis of our critique of other theories in this area for more than 10 years (Chater & Oaksford, 1990, 2001; Oaksford & Chater, 1991, 1993, 1995, 1998, 2001). The subjective probabilities that figure in our models are derived from prior knowledge stored in long-term memory, which is also the source of the parameters of Schroyens and Schaeken's validating search procedure. The intuition behind the conditional probability model, the Ramsey (1931) thought experiment, makes this explicit (see also, Evans, Handley, & Over, in press). According to Ramsey, people evaluate a conditional by first adding the antecedent (p) to their stock of beliefs, making minimal adjustments to establish consistency, and then evaluating the support this revised set of beliefs provides for the consequent (q). For repeatable events this will involve searching memory for counterexamples (not-q) and positive instances (q). For nonrepeatable events it will involve establishing other types of evidential relationships. As we pointed out early in this article, the normative literature seems to have converged on the view that such a model is required to explain everyday indicative conditionals (Adams, 1966, 1975; Bennett, 1995; Edgington, 1995). Thus, if psychological theories of abstract reasoning tasks have any aspirations to generalize to real, everyday conditional inferences, it is a good thing that some consensus is emerging on the importance of probabilistic prior knowledge.

[7] Schroyens and Schaeken would probably argue that for abstract material, with no prior knowledge, people should assume that the ability of any contrast-class member to support the AC inference in Examples 2a and 2b should be equal. However, surely a much more informative prior probability distribution would be to go with their prior knowledge that only one member (or perhaps a few) of a contrast set has this ability; thus, most of them probably do not. This also seems more consistent with Schroyens et al.'s (2000) own view of the role of prior knowledge (see *MP Inference Rates* section).

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