

COMMENTS

Holography Does Not Account for Goodness: A Critique of van der Helm and Leeuwenberg (1996)

Christian N. L. Olivers
University of Birmingham

Nick Chater and Derrick G. Watson
University of Warwick

P. A. van der Helm and E. L. J. Leeuwenberg (1996) outlined a holographic account of figural goodness of a perceptual stimulus. The theory is mathematically precise and can be applied to a broad spectrum of empirical data. The authors argue, however, that the account is inadequate on both theoretical and empirical grounds. The theoretical difficulties concern the internal consistency of the account and its reliance on unspecified auxiliary assumptions. The account also makes counterintuitive empirical predictions, which do not fit past data or the results of a series of new experimental studies.

Figural goodness is typically used to refer to the salience or strength of a perceptual regularity. The notion has strong intuitive roots. For example, most people will feel that Figure 1B is “better” than Figure 1A. The goodness of a regularity can also be operationalized in several ways. For instance, bilateral symmetry (see Figure 1B) is generally rated as “good” (Hamada & Ishihara, 1988; Masame, 1986), is responded to quickly (Baylis & Driver, 1994; Pomerantz, 1977; although see Olivers & van der Helm, 1998), is easily discriminated from random figures (e.g., Wagemans, Van Gool, & d’Ydewalle, 1991), is relatively noise resistant (Barlow & Reeves, 1979; Jenkins, 1982), and is well remembered (Attneave, 1955). Henceforth, we make the standard assumption that *goodness* refers not just to an intuitive notion but to a theoretically interesting perceptual property underlying, perhaps in a complex way, the measures we have mentioned. The study of goodness, therefore, seeks to address fundamental questions in perception: Why are some structures salient and rapidly detected, whereas other structures appear weak and difficult to detect?

Traditionally, the *transformational* approach to goodness has been most influential: Stimulus regularities are viewed as revealed by invariance under group-theoretic transformations over the stimulus. For example, the mirror symmetric Figure 1B remains in-

variant under a “flip” transformation around its vertical axis, whereas Figure 1C remains invariant under rotations in steps of 90°. More invariant transformations lead to greater perceived goodness (Garner, 1974; Palmer, 1982, 1983, 1991).

Recently, however, van der Helm and Leeuwenberg (1996) proposed an important alternative approach, based on a specific coding-theoretic account of perception, structural information theory (SIT; Leeuwenberg, 1969; Leeuwenberg & Buffart, 1983). SIT was developed to explain how the perceptual system chooses between rival perceptual organizations: The perceptual system chooses the simplest organization, that is, that with the shortest code in the SIT coding language. It is important to note that van der Helm and Leeuwenberg (1996) affirmed this account but argued that the goodness of the resulting organization cannot also be determined by simplicity. Their most important counterexample is that mirror symmetry is better than twofold repetition, even though these are equally simple (e.g., Bruce & Morgan, 1975; Corballis & Roldan, 1974; see Figure 2). Instead, van der Helm and Leeuwenberg (1996) proposed that goodness is determined by a new measure: *weight of evidence* (W), which, they argued, captures a wide range of empirical data.

W is based on a novel mathematical analysis of regularities in 1-D pattern codes, dubbed the *holographic* approach (van der Helm & Leeuwenberg, 1991, 1996). However, here, we argue that the holographic approach does not capture the broad regularities concerning goodness. It fails theoretically because of the insecure link between mathematical theory and psychological predictions as well as because of problems of internal consistency. Moreover, the account makes counterintuitive predictions, which we experimentally disconfirm. This article has three main sections. The Holographic Theory of Goodness: From SIT to Weight of Evidence section describes van der Helm and Leeuwenberg’s (1996) account. The Evaluating the Holographic Theory of Figural Goodness section outlines theoretical and empirical difficulties. The General Discussion section summarizes and considers the future of the holographic approach to goodness.

Christian N. L. Olivers, School of Psychology, University of Birmingham, Birmingham, United Kingdom; Nick Chater and Derrick G. Watson, Department of Psychology, University of Warwick, Coventry, United Kingdom.

This work was partially supported by grants from the Medical Research Council, United Kingdom, and by European Commission Grant RTN-HPRN-CT-1999-00065 to Nick Chater. We thank Johan Wagemans for many useful comments and Johan Hulleman for fruitful discussions.

Correspondence concerning this article should be addressed to Christian N. L. Olivers, who is now at the Department of Cognitive Psychology, Vrije Universiteit, Van der Boechorststraat 1, 1081 BT Amsterdam, the Netherlands. E-mail: cnl.olivers@psy.vu.nl

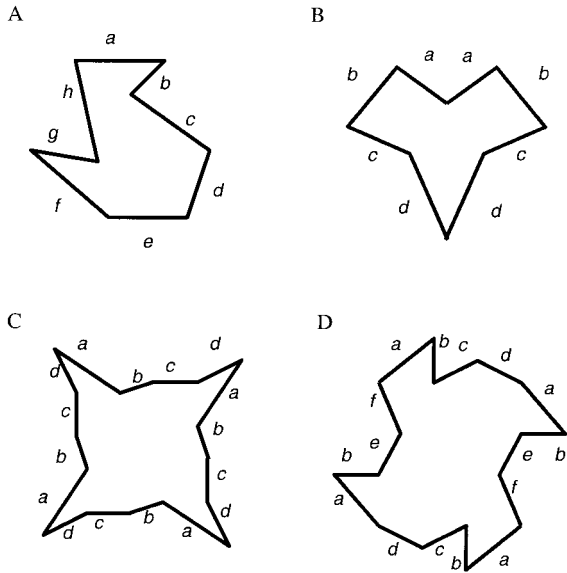


Figure 1. Examples of different regularities. A: Random polygon. B: Bilateral symmetry. C: Fourfold rotational symmetry. D: Twofold rotational symmetry with alternation. The shapes have been assigned simplified structural-information-theory-style codes with codes *abcdefgh* (A), *abcdedcba* (B), *abcdabcdabcdabcd* (C), and *abcdabefabcdabef* (D).

Holographic Theory of Goodness: From SIT to Weight of Evidence

There are three key elements in the holographic approach to goodness. The first is SIT, mentioned above (Leeuwenberg, 1969; Leeuwenberg & Buffart, 1983). The second is the holographic theory itself (van der Helm, 1988; van der Helm & Leeuwenberg, 1991, 1996), which was intended to provide a rigorous mathematical foundation for SIT. The third element is the weight of evidence, *W*, based on the holographic account, which is van der Helm and Leeuwenberg's (1996) measure of goodness. We outline these elements below.

SIT

The perceptual stimulus can be organized in a limitless number of ways. But typically, just one or at most a small number of organizations are perceptually apparent (e.g., Hatfield & Epstein, 1985; Leeuwenberg & Boselie, 1988; Pomerantz & Kubovy, 1986). By what principle is the dominant perceptual organization chosen? One influential viewpoint, which can be traced to Mach (1886/1897) and Gestalt psychology (Koffka, 1935/1963), is the *simplicity principle*: The perceptual system prefers the simplest organization that it can find (e.g., Attneave, 1954; Chater, 1996, 1999). Spelling out this view requires measuring simplicity. One direct way of doing this is to devise a perceptual coding language and to identify perceptual organizations with codes that embody the regularities in those organizations. Thus, if the organization postulates that the stimulus is symmetrical, the corresponding code would use a symmetry predicate or operator. This approach is known as coding theory (Restle, 1979; Simon, 1972), of which SIT

is the most important example (Leeuwenberg, 1969; Leeuwenberg & Buffart, 1983).

The SIT visual coding language is initially defined over 1-D sequences of symbols. These 1-D sequences are then used to describe regularities in 2-D visual patterns. Figure 1 shows how symbols are assigned to the contour elements. For instance, we can assign the symbol sequence *abcdefgh* to the irregular contour of Figure 1A. This figure has eight contour elements, and this code can thus be assigned an informational load of $I = 8$. Similarly, we can assign a symbol sequence to Figure 1B, with identical elements receiving identical symbols. Tracing the contour, we show that the symbol series becomes *abcdedcba*, which can be simplified by capturing its bilateral symmetry in one of SIT's coding rules, the symmetry rule (S-rule): $S[(a)(b)(c)(d)]$. In this code, there are only four perceptual elements left, hence $I = 4$. (Note that SIT counts only the remaining number of perceptual elements; i.e., it does not include the S symbol in its perceptual load; van der Helm, van Lier, & Leeuwenberg, 1992. Note also that, purely for clarity, we frequently use spaces to parse codes into chunks.) Because the symmetry code is simpler (i.e., shorter) than the raw code, it is perceptually preferred.

Figures 1C and 1D demonstrate two more SIT coding rules: the iteration rule (I-rule) and the alternation rule (A-rule). In Figure 1C, the contour elements *a, b, c,* and *d* are repeated three times. The resulting sequence, *abcdabcdabcdabcd*, is thus coded as the threefold repetition $4 * (abcd)$, by the I-rule. According to SIT, $I = 4$ because there are four symbols remaining in the code (again, the scalar, 4, and the operator, $*$, are not counted). The raw symbol code for Figure 1D is *abcdabefabcdabef*, which the A-rule codes as $\langle(ab)\rangle\langle(cd)(ef)(cd)(ef)\rangle$, with $I = 10$. [Note here how *(ab)* alternates with *(cd)* and *(ef)*.]

Finally, within SIT, codes can combine hierarchically. For example, the alternation $\langle(ab)\rangle\langle(cd)(ef)(cd)(ef)\rangle$ of Figure 1D can be simplified by coding the repetition, leading to $2 * (\langle(ab)\rangle\langle(cd)(ef)\rangle)$, with $I = 6$. The repetition in the code corresponds directly to the repetition in the contour of the polygon. In practice, the I-, S-, and A-rules (called the *ISA-rules*) have been the most popular rules in SIT.

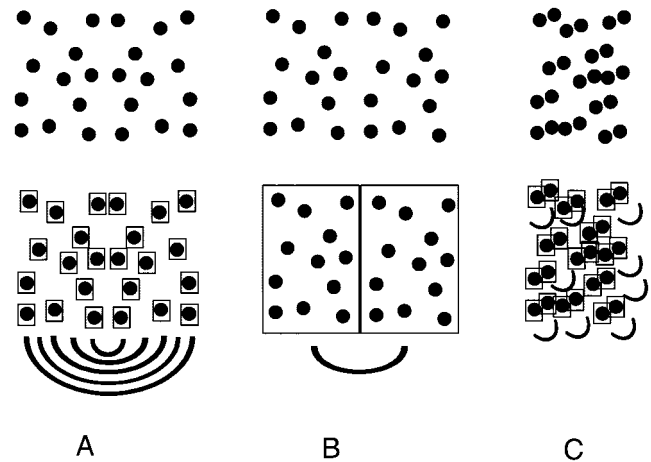


Figure 2. Three different regularities and the structures assigned to them by holography. A: Bilateral symmetry, point structure. B: Repetition, block structure. C: Glass pattern, point structure.

Holographic Approach

A fundamental issue in building any coding language is which regularities to include and which not to include. Early statements of SIT (e.g., Leeuwenberg, 1969; Leeuwenberg & Buffart, 1983) did not provide any justification for why the ISA-rules were allowed, whereas a vast range of other conceivable rules were not. Without such justification, the constructs of SIT were somewhat ad hoc.

The holographic approach was developed to address exactly this issue, by providing a firm theoretical foundation for the choice of regularities in a coding language (van der Helm, 1988; van der Helm & Leeuwenberg, 1991, 1996, 1999). Van der Helm and Leeuwenberg (1991, 1996, 1999) argued that a visual regularity must be accessible for the visual system to detect it. Specifically, the “regularity and hierarchy in a code of a pattern should correspond directly to regularity and hierarchy in the pattern itself” (van der Helm & Leeuwenberg, 1991, p. 167). Accessibility is determined by two subcriteria, *holography* and *transparent hierarchy*, that, from all possible rules, eliminate those that are perceptually inaccessible. According to van der Helm and Leeuwenberg (1991, 1996), there are essentially only three regularities that are both holographic and transparent: the ISA-rules. Thus, holography provides a formal justification for SIT’s coding rules.

Holography

Intuitively, a holographic regularity in a code corresponds directly to a regularity in a pattern, in a holistic way. That is, if a particular piece of code describes a regularity, then every single element of that code should be involved in describing that very same regularity. This concept is formalized as follows. The regularity in a symbol series can be expressed in so-called *identity chains*. For instance, the bilateral symmetry *abcd dcba* can be described by the identity chain $\{(1) = (8), (2) = (7), (3) = (6), (4) = (5)\}$, in which the numbers 1–8 signify the position of the symbol in the sequence (with the first symbol, *a*, being identical to the eighth symbol; the second, *b*, to the seventh symbol; etc.). Note also that the group of identities can be split into subchains, each of which embodies the very same regularity, namely a bilateral symmetry. For example, the subchain $\{(1) = (8), (3) = (6)\}$ corresponds to the symmetry in *a_c_c_a* (underscores denote arbitrary elements). It can be shown that for a bilaterally symmetric code, all its identity chains also refer directly to bilaterally symmetric codes. Hence, bilateral symmetry is a *holographic* regularity: Its symmetry is omnipresent in its identity chains. Not all regularities are holographic. Van der Helm and Leeuwenberg (1996) considered the sequence *kpf kfp*, which is described by the identity chain $\{(1) = (4), (2) = (6), (3) = (5)\}$. Its regularity could be captured in a general SIT-style rule, $B(x,y,z)$ in which *x*, *y*, and *z* are repeated but *y* and *z* are swapped from one chunk to the other. Van der Helm and Leeuwenberg noted that this regularity is not holographic: The subchains $\{(1) = (4), (2) = (6)\}$, $\{(2) = (6), (3) = (5)\}$, and $\{(1) = (4), (3) = (5)\}$ do not all correspond to one and the same regularity; that is, they are not all typical instances of the *B-rule*.

Transparent Hierarchy

The holographic rules are further sifted by the second accessibility criterion, transparent hierarchy. We noted earlier that SIT’s

coding rules can be hierarchically combined: For example, *abab baba* can be coded $S[(a)(b)(a)(b)]$ and simplified to $S[2 * ((a)(b))]$. Now, the hierarchy in this code is *transparent*, because the higher order repetition $2 * ((a)(b))$ corresponds directly to the *abab* repetition in the raw symbol sequence. More generally, the idea is that a hierarchy is transparent if a regularity described at any level in the hierarchy corresponds directly to the same type of regularity at the bottom of the hierarchy, that is, the raw symbol sequence. Thus, the regularity in the code is transparent in the raw symbol sequence.

Not all regularities show transparent hierarchy. For example, van der Helm and Leeuwenberg (1996) argued that the holographic *M-rule*, which codes the regularity in *ara bsb ctc* as $M[(a)(b)(c),(r)(s)(t)]$, is not transparent. Consider the sequence *ara brb ctc*, which is coded $M[(a)(b)(c), 2 * (r),(t)]$. The higher order repetition $2 * (r)$ does not correspond to a repetition (e.g., *rr*) in the symbol sequence, and so the *M-rule* is not transparent. According to van der Helm and Leeuwenberg (1996), only the ISA-rules are both holographic and transparent (although see our evaluation below).

From Holography to Weight of Evidence

Van der Helm and Leeuwenberg (1996) went even further. Holography not only offered a reason for the priority of certain visual regularities over others but also explained some of the most puzzling goodness phenomena. Figure 2 illustrates a particularly important case. Figure 2A shows a random dot pattern that has been copied and mirror reversed, whereas Figure 2B shows a repetition by translation of the same subpattern. Bilateral symmetry is generally considered better than repetition (Baylis & Driver, 1994; Bruce & Morgan, 1975; Chipman, 1977; Chipman & Mendelson, 1979; Corballis & Roldan, 1974; Julesz, 1971; Kahn & Foster, 1986), although both patterns are intuitively equally simple. In line with this intuition, the two patterns have the same code length in SIT. Figure 2A could be coded as $S[(a)(b)(c)(d) \dots (l)]$ and Figure 2B as $2 * (abcd \dots l)$, so that informational load, *I*, is 12 for both patterns (the 12 dots in the subpattern have been assigned symbols *a–l*).

Figure 2C also shows a repetition of the basic dot pattern, but the translation is over a much shorter distance than in Figure 2B. The pattern now resembles a Glass pattern (Glass, 1969). Unlike the translation in Figure 2B, and even though they are equally simple, Glass translations usually induce very strong percepts, which are resistant to relatively high levels of noise (Glass & Switkes, 1976; Maloney, Mitchison, & Barlow, 1987). Empirically, then, in terms of goodness, Glass patterns appear to lie closer to bilateral symmetries than to long-distance translations.

At first, one might expect that within SIT Figure 2C would be coded in the same way as Figure 2B, that is, $2 * (abcd \dots l)$. However, van der Helm and Leeuwenberg (1996, p. 451) suggested that Glass patterns are better coded using the *A-rule*. They proposed that for Glass patterns the visual system codes the position of one member of each dot pair and then makes these positions alternate with a “Glass relationship.” Thus, in Figure 2C, the initial subpattern is coded as *abcd ... l*, as usual. The Glass relationship *G* with the second member of each dot pair is then introduced, resulting in *aGbGcGdG ... lG*, which is coded as the alternation $\langle (a)(b)(c)(d) \dots (l) \rangle \langle (G) \rangle$.

The informational load is the same for each of these regularities, but their goodness differs. Van der Helm and Leeuwenberg (1996) therefore proposed that goodness must be determined by the structure of the regularities, rather than by their simplicity. Crucially, van der Helm and Leeuwenberg (1996) argued that bilateral symmetry and repetition have a fundamentally different structure. They claimed that whereas bilateral symmetry has a *point* structure, repetition has a *block* structure (see the bottom panel of Figure 2): a distinction that can be clarified by considering how regularities grow. Consider the symmetry *abcddcba*, with the identity chain $\{(1) = (8), (2) = (7), (3) = (6), (4) = (5)\}$. To modify this symmetry in compliance with holography, one must add or remove identities without disturbing the overall regularity. For instance, taking away $\{(1) = (8)\}$ leaves the symmetric *bcddeb*; adding $\{(0) = (9)\}$ results in the symmetry *zabcdcbaz*. Because the regularity extends “point-by-point,” van der Helm and Leeuwenberg (1996) said that symmetry has a point structure (see Figure 2A). This point structure implies that every identity in the holographic chain embodies the bilateral symmetry—thus providing considerable weight of evidence for the overall regularity.

Now consider the repetition *abc abc abc*. There are two possible identity chains for this repetition. The first is $\{(1) = (4), (2) = (5), (3) = (6), (4) = (7), (5) = (8), (6) = (9)\}$ and has a point structure. However, this chain is not holographically extendible or reducible: Removing, for example, $\{(1) = (4)\}$ results in *_bc _bc abc*, which is not a repetition according to van der Helm and Leeuwenberg (1996). Adding the identity $\{(7) = (10)\}$ leads to *abc abc abc a*, which also violates the overall repetition. The second type of identity chain divides the repetition into blocks: $\{(1\ 2\ 3) = (4\ 5\ 6), (4\ 5\ 6) = (7\ 8\ 9)\}$. This type of identity chain can be holographically extended or reduced by one identity, and hence by one block, at a time [e.g., adding $\{(7\ 8\ 9) = (10\ 11\ 12)\}$ results in *abc abc abc abc*]. Because repetition can only extend a block at a time, van der Helm and Leeuwenberg referred to repetition as having a block structure. Crucially, because only the identities between the blocks provide evidence for the regularity, there is much less weight of evidence for its presence (see Figure 2B).

Thus, a regularity with a point structure, like symmetry, has greater weight of evidence than a regularity with a block structure, like repetition. This suggests the number of identities in a code may determine the goodness of a regularity. Specifically, van der Helm and Leeuwenberg (1996) quantified the weight of evidence W for a code as the ratio of the number of identities in a regularity (E) and the number of elements in the raw symbol code (n):

$$W = E/n. \tag{1}$$

Thus, in the symmetry in Figure 2A, there are $E = 12$ identities and $n = 24$ raw elements (i.e., the dots), yielding $W = 12/24 = 1/2$. By contrast, the repetition in Figure 2B has a weight of evidence of only $1/24$ because the pattern is organized into two blocks of 12 elements, linked by only one identity. Hence, W explains why bilateral symmetry is so much better than repetition.

Now, consider the goodness of Glass patterns. Alternations, like bilateral symmetries, have a point structure. The alternation of Figure 2C contains 11 identities, and the goodness therefore is $W = 11/24$, which is almost as good as bilateral symmetry. This explains why Glass patterns are generally so strong.

Finally, consider rotational (or centric) symmetry. This can have high goodness, although typically less than bilateral symmetries

(e.g., Kahn & Foster, 1986; Palmer & Hemenway, 1978; Royer, 1966). The ISA-rules do not directly provide for rotational symmetries, but van der Helm and Leeuwenberg (1996) suggested that rotations are “repetitions in polar co-ordinates” (pp. 429, 440). Thus, rotations get a block structure and are hence less good than bilateral symmetries.

Note, however, that holography still envisages a crucial role for the simplicity principle in perceptual organization. Pattern representations are still selected solely on the basis of their simplicity. But the goodness of the selected pattern is determined by W (van der Helm & Leeuwenberg, 1996, p. 444). This ingenious line of reasoning promises to reconcile the simplicity account of perceptual organization with apparent differences in goodness, such as between symmetry and repetition, that simplicity alone appears unable to explain. In addition, van der Helm and Leeuwenberg (1996) argued that this dissociation also accounts for many other goodness phenomena, many of which we discuss below. Furthermore, holography appears to provide rigorous formal foundations for its key psychological concepts (van der Helm, 1988; van der Helm & Leeuwenberg, 1991). This, according to the authors, creates a “win–win situation” relative to other approaches, such as Palmer’s (1982, 1983) transformational approach and Wageman et al.’s (1991; Wagemans, Van Gool, Swinnen, & Van Horebeek, 1993) bootstrap model, which suffer from making psychological assumptions for which there exists no formal foundation (van der Helm & Leeuwenberg, 1999, p. 624). However, in this article, we demonstrate that the formalizations of van der Helm and Leeuwenberg (1991) do not underpin the holographic approach to figural goodness outlined in van der Helm and Leeuwenberg (1996). We argue that point and block structures are arbitrarily interchangeable and crucially depend on the specific spatial mapping of a regularity onto a symbol code. Once this mapping is specified, holography becomes redundant in explaining goodness. Even if point and block structures are taken as given, the theory leads to incorrect and sometimes contradictory predictions. In other words, holography itself suffers from making psychological assumptions for which there exist no formal foundations.

Evaluating the Holographic Theory of Figural Goodness

In this section, we systematically analyze van der Helm and Leeuwenberg’s (1996) most important claims. We focus on those issues that have not previously been critically examined in detail (e.g., in van der Helm & Leeuwenberg, 1999, and Wagemans, 1999).

Before we present our critique, however, a few notes are in order. First, it is worth pointing out that van der Helm and Leeuwenberg’s (1996) account does not relate directly to our knowledge of the functional and neural mechanisms involved in vision. Instead, they concentrated on an abstract and mathematical level of representation, while ignoring possible effects of attention and spatial frequency as well as of the different operationalizations of goodness (with sometimes different results). This may seem problematic, as it would be surprising if such factors did not substantially affect performance. Any abstract theory of vision, in the end, requires detailed augmentation with constraints imposed by the machinery of the visual system. Van der Helm and Leeuwenberg were aware of this concern but believed that an abstract account might, nonetheless, provide deep insights that would be

missed by “biases” at the processing level (van der Helm & Leeuwenberg, 1996, pp. 431–436). As this may be true, we believe that it would be inappropriate at this stage to open lines of attack on the level of the exact implementation and operationalization of the theory. As a result, our critique often makes use of equally abstract counterexamples, which we do not claim to be any more neurally or functionally plausible, unless stated otherwise. They serve just to demonstrate inconsistencies. Second, it might be argued that no single measure of goodness can be adequate because goodness is operationalized with many measures (reaction times [RTs], error rates, sensitivity, and noise resistance), few of which are linearly related. Consequently, holography’s W , being a single measure, could be immediately falsified.¹ However, we believe it would be unfair to criticize van der Helm and Leeuwenberg for attempting to bring some unification in the diverse research field on goodness. Instead, van der Helm and Leeuwenberg’s theoretical unification requires, we suggest, just that W is monotonically, but not necessarily linearly, related to the different operationalizations of goodness.

Claim 1: Bilateral Symmetries and Alternations Have Point Structures, Whereas Repetitions and Rotations Have Block Structures (van der Helm & Leeuwenberg, 1996, pp. 429, 438–440; 1999, p. 622)

The assignment of point and block structures to specific regularities is central to the holographic account. However, this assignment is, on closer inspection, not determined by the theory. The first problem is the order of the symbols in a series—a point raised earlier by Wagemans (1999). Consider the symmetrical pattern in Figure 3A. Following SIT, each line segment is labeled with a symbol. Corresponding line segments are given the same label, reducing the pattern to an array of eight symbols, displayed in two vertical columns. We now face a critical question: How should this 2-D array be mapped onto a 1-D symbol string? Only when this mapping has been achieved can SIT, holography, and weight of evidence be applied. Yet, there appear to be many mappings available. Van der Helm and Leeuwenberg (1996) were implicitly committed to the mapping shown in Figure 3A: The 1-D string is defined to start at the upper left corner and finish in the upper right corner, following a U-shaped path. The resulting symbol string codes the bilateral symmetry as $abcdcda$, which indeed results in a holographic point structure. But, if we defined the 1-D string to trace a mirror-reversed N (see Figure 3B), we would obtain the alternative path $abcdabcd$, which results in a holographic block structure. Thus, whether mirror symmetry has a point or block structure is not predicted by the holography; it is determined by an implicit assumption about the choice of mapping.

The same point applies to repetitions. Using the mirror-reversed N route (which was rejected for mirror symmetry), we obtain the path preferred by van der Helm and Leeuwenberg (1996), $abcdabcd$ (see Figure 3D), and we assign repetition a block structure. But with the U route, the resulting code would be $abcdcda$ (see Figure 3E), and repetition would be assigned a point structure. This point is further illustrated by a fourth rule, the *T-rule*. Van der Helm and Leeuwenberg (1991) allowed that the T-rule is both holographic and transparent (pp. 196–197). It codes a repetition as $T[(a)(b)(c)(d)]$, as illustrated by the Z-shaped route in Figure 3F. However, in a subsequent article, van der Helm and Leeuwenberg

(1996) too quickly dismiss it as “completely accounted for by repetition as described by the I-rule” (p. 442) and do not therefore include the T-rule in any of their analyses. Yet, in their account of figural goodness, the T-rule takes on considerable theoretical significance: It describes repetitions using a point structure because its identity chains consist of individual pairs of pattern elements (an easy test shows how identities can be removed or added without destroying the T-regularity). Note, too, that symmetry can also be coded using the T-rule, using the same Z route (see Figure 3C).²

The assignment of codes, and hence point and block structures, is also arbitrary for the rotational symmetries illustrated in Figures 3G and 3H. We could, as van der Helm and Leeuwenberg (1996) suggested, code the rotation as a repetition (using the I-rule), leading to a block structure. However, we could also invent a new *P-rule* (“P” for point symmetry), which follows a route through the center of symmetry and hence codes this particular rotation as $P[(a)(b)(c)(d)]$. The P-rule is both holographic and transparent (note its similarity to the T-rule) but results in a point structure, $aabbccdd$, rather than the block structure proposed by van der Helm and Leeuwenberg.

The same points apply to the growth of regularities. Whereas van der Helm and Leeuwenberg (1996) suggested that repetitions grow blockwise and bilateral symmetries grow pointwise, the examples in Figure 4 show this can easily be reversed. It all depends on the direction in which we add the elements. With 2-D

¹ We thank an anonymous reviewer for suggesting this point.

² One case in which the U route may be preferred over the N route for symmetries (and vice versa for repetitions) is when line segments and their angles are coded separately. For instance, in the bilateral symmetry of Figure 3A, if we start at segment a on the left half and trace the contour toward b , we need to turn x° . Likewise, b and c are at an angle of y° to each other, and so on. Taking the N route would mean that in the second half of the pattern angles receive an opposite sign. For instance, if we start at segment a on the right and follow the route down to b and c , we need to turn $-x^\circ$ and $-y^\circ$, respectively. Van der Helm and Leeuwenberg (1996) may argue that the visual system is unable to match equal angles of opposite sign, which would mean that the N route is unable to encode the regularity. Only the U route leads to angles equal in sign. The same can be shown for the N route in repetitions. However, there are a number of problems with this argument. First, in case of repetitions, the N route may work fine for the stimuli in Figure 3, in which the two pattern halves are separated. However, we can easily close the contours, by connecting the top and bottom segments, to form an outline polygon. The N route would now have to break away from the fully connected contour, resulting in a violation of van der Helm and Leeuwenberg’s (1996) definition of spatial contiguity (see the General Discussion section). Second, were such break-aways allowed nevertheless, then the most straightforward way of coding would be via the T-rule for symmetries and repetitions alike—thus still making any point and block structures arbitrary. Third, we can think of regularities that are easily perceivable but are difficult to explain if angles of opposite sign but equal magnitude are treated as completely different—such as the glide (Hardonk, 1999; see also Wolfe & Friedman-Hill, 1992). Finally, compare the curved stimuli \smile (and \frown). It is difficult to see what the angles are here, unless we impose a limited resolution on the curvature: That is, we assume that the visual system sees the curvature as existing of many tiny line segments at many different angles. However, contrary to what has been found (e.g., Pomerantz, 1977), this would mean an extremely low goodness for the \smile repetition (because of the high n in the W ratio) compared with the \smile symmetry.

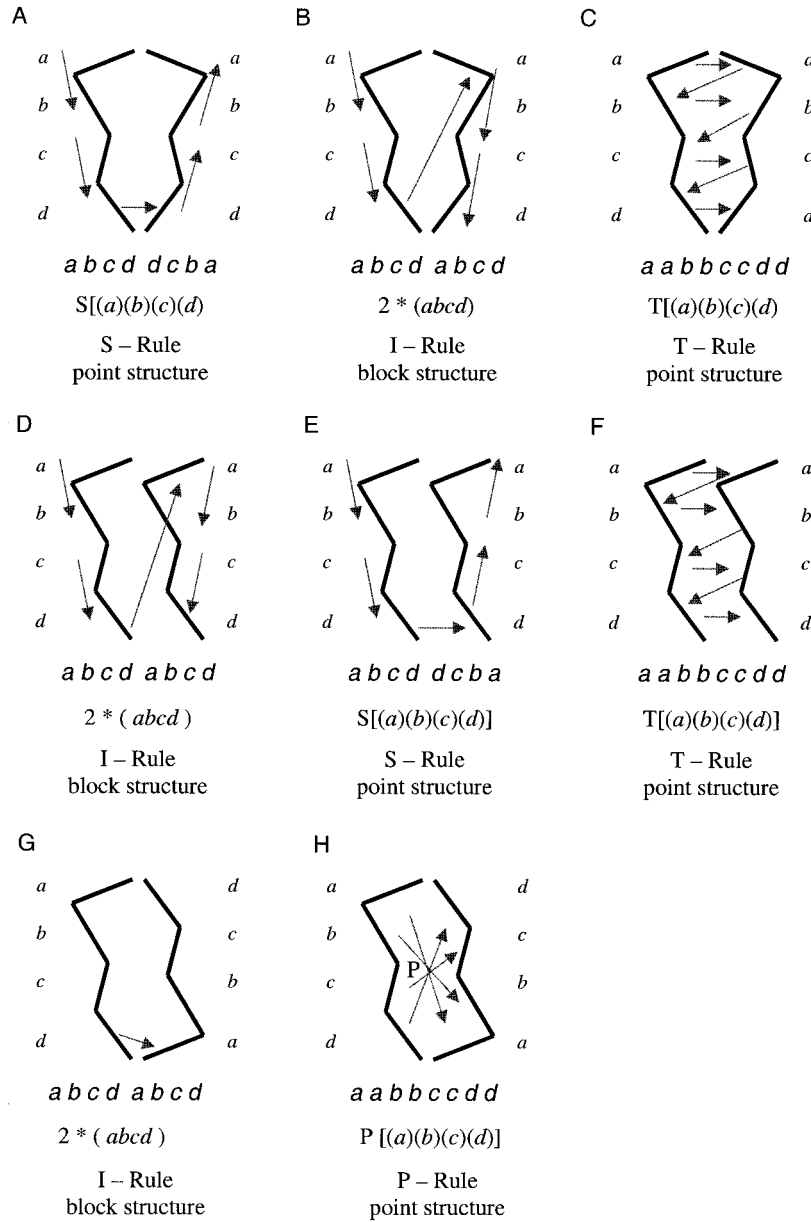


Figure 3. Bilateral symmetry (A, B, and C), repetition (D, E, and F), and rotation (G and H), plus their possible structures (block or point) as imposed by different coding rules (S, I, T, and P). The I-rule results in block structures, the S-, T-, and P-rules in point structures. See the text for details on these rules.

or 3-D patterns, multiple growth directions are possible, resulting in different structures. However, by dealing only with 1-D symbol strings, holography obscures these possibilities.

Van der Helm and Leeuwenberg (1996) might raise two objections to this. First, they may argue that they do have a criterion for ordering symbols: *spatial contiguity* (van der Helm & Leeuwenberg, 1996, p. 443). Spatial contiguity has not been rigorously defined (see the General Discussion section), but van der Helm and Leeuwenberg proposed that the visual system may follow a particular path through the pattern elements, resulting in a certain symbol order. However, there appears no rationale based on spatial contiguity of why the visual system should jump back to the top of

the pattern when coding the second half of a repetition (the N route; see Figure 3D) but stay at the bottom when coding the second half of a bilateral symmetry (the U route; see Figure 3A). Indeed, only the T-rule seems to provide a consistent path (the Z route) for both regularities, but it results in a point structure for symmetry and repetition. Spatial contiguity is especially problematic in random dot patterns, in which there are no cues concerning which route to follow. In fact, in a reasonably dense dot pattern (e.g., Julesz, 1971), millions of routes are possible, some resulting in block structures and some in point structures. Furthermore, if spatial contiguity—whatever its definition—is so theoretically critical, there is the danger that it may explain goodness without

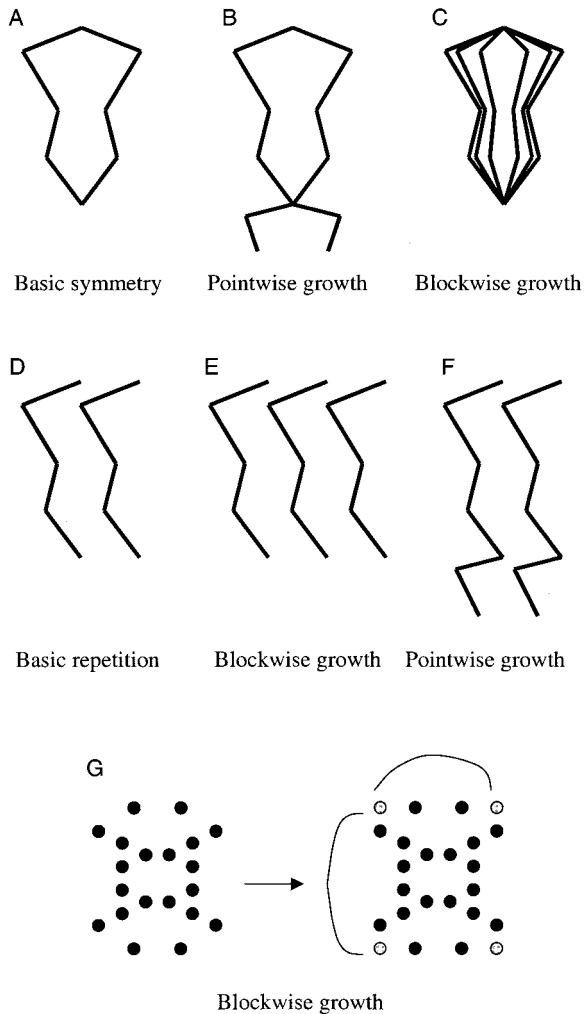


Figure 4. Bilateral symmetry (A, B, and C) and repetition (D, E, and F), plus their possible directions of growth (blockwise or pointwise). G shows how a multiple symmetry needs the addition of more than one identity to grow while still maintaining its status as a multiple symmetry.

the necessity for holography or transparent hierarchy. We shall see that spatial contiguity is a recurring problem for holography (see also Wagemans, 1999).

Even under a consistent spatial-contiguity criterion, there is no a priori reason for why repetitions could not grow pointwise within holography. Consider the pattern $abc\ abc\ abc$, with the code $3 * (abc)$, by the I-rule. Now consider $abc\ abc\ abc\ a$. There is a holographic rule (again, mentioned in van der Helm & Leeuwenberg, 1991) that can fully capture the regularity in this pattern; call this the J-rule. The J-rule codes $abc\ abc\ abc\ a$ as $J[(a), 3 * (bc)]$. More generally, the J-rule takes on the form $J[(x), (y)]$, meaning that any y will be interspersed with x . Notice how this pattern is a pointwise extension of the iteration in the first example. The pointwise extension is further illustrated by the next step: $abc\ abc\ abc\ ab$, which the J-rule codes as $J[(ab), 3 * (c)]$. The final step, $abc\ abc\ abc\ abc$, brings us back to the—now extended—iteration $4 * (abc)$, as coded by the I-rule. However, notice that the I-rule is

no more than a special instance of the J-rule, which would code this pattern $J[(abc), 4 * ()]$ (i.e., without the second parameter).³

A second objection may be that the onefold repetition as described by the T-rule (e.g., $aabbccdd$) is not representative of how the visual system codes repetitions in general. Van der Helm and Leeuwenberg (1996, pp. 438–439) argued that repetitions are infinitely and holographically extendible (e.g., from $abcd\ abcd\ abcd$ to $abcd\ abcd\ abcd\ abcd$, etc.), which, van der Helm and Leeuwenberg said, is not possible with the T-rule and can be achieved only if the visual system assumes a block structure. Thus, they might argue that the onefold repetition too must be coded as $abcd\ abcd$, even though the T-rule applies in principle. Note, however, that in the present context, this argument is circular. It requires using constraints on what the visual system can detect (the fact that it can encode multiple repetitions) as a criterion to choose holographic coding rules, whereas the holographic coding rules are intended to explain the constraints on what the visual system can detect. Leaving aside circularity, the special status of multiple repetition causes problems when equally applied to multiple symmetry. Figure 4G shows a multiple (twofold) symmetry. Making this symmetry grow, while maintaining its status of a multiple symmetry, requires adding two identities simultaneously. But, this is not allowed within holography, in which regularities should grow by one identity at the time. By analogy to multiple repetitions, we would have to assume a block structure for multiple symmetries (which would incorrectly imply low goodness).

In summary, there is no criterion for mapping from 2-D arrays to 1-D symbol strings. Hence, whether particular regularities are assigned point or block structures is arbitrary and cannot help explain goodness. Growth of symbol series is not dictated by holography but depends entirely on which regularity is believed to be relevant and in which order this regularity is coded. Below, however, we assume, for argument's sake, that goodness is based on point and block structures, as van der Helm and Leeuwenberg (1996) assigned them. We show that the account still faces difficulties.

³ To our knowledge, the J-rule is both holographic and transparent. However, the J-rule may be dismissed on the basis of the hierarchical correspondence not being completely unambiguous (van der Helm & Leeuwenberg, 1991). For instance, if we encode $abcabcabca$ as $J[(a), 3 * (bc)]$, then the higher order $3 * (bc)$ repetition corresponds to the lower order $3 * (abc)$ repetition as well as to the $3 * (bca)$ repetition in the basic symbol sequence. Hence, according to van der Helm and Leeuwenberg (1991), the J-rule is not completely transparent. But, this is by no means clear. First, the ambiguity pertains only to where in the code the repetition starts; that is, it can be shifted by one symbol position. In contrast, as we understand it, the hierarchy itself is unambiguously transparent, as the higher order regularity corresponds to the same kind of regularity at a lower level (see van der Helm & Leeuwenberg, 1991, p. 193). Second, the ambiguity as to where exactly in a code the extraction of a particular higher order regularity should start occurs in many other places in holographic coding. An example is given by van der Helm and Leeuwenberg (1996, pp. 447–448), who demonstrated the ambiguity in the hierarchy of threefold mirror symmetries. Threefold symmetries can be hierarchically coded as one global bilateral symmetry combined with two local bilateral symmetries but also as three local symmetries (all with the valid S-rule). Even if the global symmetry is defined, there is ambiguity as to where exactly the local symmetry should start. However, none of these ambiguities serve as a criterion to dismiss the S-rule as being nontransparent.

Claim 2: Goodness Is Independent of Simplicity (van der Helm & Leeuwenberg, 1996, p. 444)

The holographic approach uncouples goodness from simplicity. The visual system selects the simplest code, but the goodness is measured by *W*. However, this uncoupling leads to contradictory predictions if we consider the problem of finding a pattern in noise. This is because, on van der Helm and Leeuwenberg's (1996) account, both notions can be applied to the issue, yielding different predictions.

From the perspective of holography, the noise resistance of a pattern is a monotonically increasing function of its weight of evidence, *W*. Thus, because symmetry has a much higher *W* than repetition, the mirror-reversed pattern in Figure 5 should be much more noise resistant than its repeated equivalent.

From the perspective of simplicity, however, discerning a regularity in noise involves choosing a perceptual organization. According to the simplicity principle, which van der Helm and Leeuwenberg (1996) endorsed, perceptual organizations are chosen to have the shortest SIT code (or more strictly, to minimize the closely related informational load). Here, this choice is between two relevant interpretations of a noisy pattern. On the one hand, there is a structure plus noise (S+N) interpretation, which views the pattern as, for example, exhibiting noisy symmetry or noisy repetition. On the other hand, there is the null interpretation, which imposes no structure on the stimulus. For a pattern with little noise, the S+N interpretation is preferred because it provides a briefer

description of the stimulus. In the case of symmetry or twofold repetition, for example, only half the stimulus must be described as well as the few "exception" points to which the symmetry does not apply. With very high levels of noise, by contrast, almost all points are exception points, and the code length for the S+N interpretation becomes greater than the null interpretation that does not attempt to impose any regularity. Crucially, therefore, there is a point at which the noise level becomes sufficient to overwhelm the regularity, and this point is the same for equally simple regularities. Under van der Helm and Leeuwenberg's (1996) claim that symmetry and twofold repetition are equally simple, we should thus predict that symmetry and repetition are equally noise resistant, in contradiction to the prediction from *W* above.

This contradiction is particularly problematic because the account of figural goodness is directly founded on the simplicity principle, as embodied in SIT. Thus, holography cannot be saved by rejecting the simplicity principle because this would undercut the foundations of the holographic approach. However, let us again ignore this theoretical complication and focus on the empirical adequacy of van der Helm and Leeuwenberg's (1996) preferred method of predicting resistance, based on *W*.

First, van der Helm and Leeuwenberg (1996) stated that holography correctly predicts that noise has relatively little effect on the goodness of bilateral symmetries. However, contrary to this prediction, small amounts of noise can dramatically affect bilateral symmetry detection, as long as it is placed near the axis of symmetry (Jenkins, 1982; Julesz, 1971; Tyler, Hardage, & Miller, 1995; Wenderoth, 1995). It seems that the visual system is sensitive only to bilateral symmetry if its elements lie relatively close to each other (see Wagemans, 1995, 1999, for similar arguments). Van der Helm and Leeuwenberg (1996, 1999) claimed that holography leaves room for such local biases—but it does not explain these phenomena.

Second, uncoupling goodness and simplicity gives the strange prediction that degrading a pattern can sometimes improve its goodness. Figure 6A shows a fourfold repetition *abcd abcd abcd abcd*. Its simplest code uses the I-rule, $4 * (abcd)$, with $W = 3/16$. Suppose we degrade the pattern by changing the last element, *d*, of the second and fourth chunks into arbitrary elements (*abcd abcx abcd abcy*; see Figure 6B). The simplicity criterion dictates that we now use a different rule to code the pattern, namely the A-rule, $\langle (abc) \rangle \langle (d) \rangle \langle (x)(y) \rangle$ (cf. van der Helm & Leeuwenberg, 1996, p. 449). This code contains four identities [three for the (*abc*) alternation and one for the nested (*d*) alternation], giving an improved goodness, $W = 4/16$. We degraded the pattern even further in Figure 6C by randomly changing all but one element in the second and last block. The simplest code within SIT is now $\langle (a) \rangle \langle (bcd) \rangle \langle (xyz)(pqr) \rangle$ and again $W = 4/16$.

There are many such examples. For instance, the degraded *abcdef abcdef* is twice as good as *abcdef abcdef*. The latter is a repetition [$2 * (abcdef)$] and, according to van der Helm and Leeuwenberg (1996), receives a block structure with one identity between the blocks: $W = 1/12$. But, *abcdef abcdef* needs a more elaborate code, which is captured by the A-rule: $\langle (abc) \rangle \langle (de)(ed) \rangle \langle (f) \rangle$, $W = 2/12$. Again, degrading a pattern is predicted to improve its goodness and, hence, noise resistance.

Van der Helm and Leeuwenberg (1996, p. 449) were aware of this type of counterintuitive prediction and, indeed, explicitly (and bravely) suggested that repetitions can be made better by adding

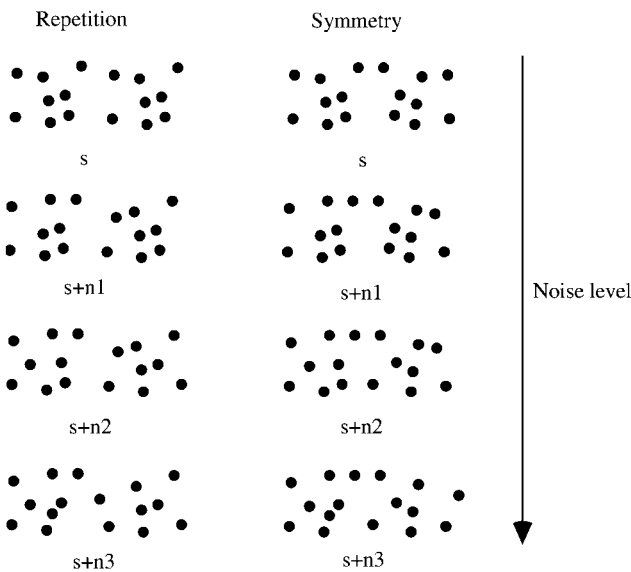


Figure 5. Repetition and symmetry. The top row shows noise-free patterns (*s*). In each successive row, the repetition and symmetrical patterns are subject to the same amount of additional noise (specifically, the same random displacements of the dots, modulo symmetry). Thus, they are each participant to successive noise levels *n*1, *n*2, *n*3, where $n_3 > n_2 > n_1$. As noise increases, the patterns become increasingly difficult to perceive, and at some point the preferred interpretation is an unstructured random field of dots. Simplicity predicts this point is the same for repetitions and bilateral symmetries. *W* predicts that this point arrives sooner for repetitions than for bilateral symmetries. As *W* is fundamentally rooted in simplicity, the holographic theory contradicts itself.

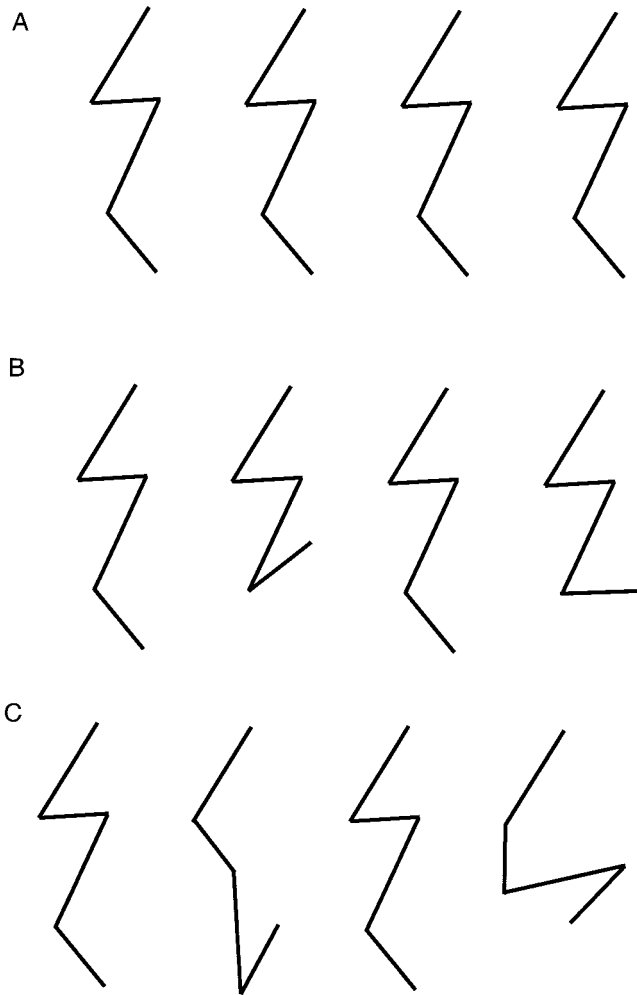


Figure 6. Stimuli of Experiment 1. A shows a perfect repetition, which is then progressively damaged in B and C. Nevertheless, holography predicts B and C to be better than A.

noise, if the noise is carefully placed at particular locations. Because there are no existing data to test this prediction, we conducted the following experiment.

Experiment 1

Method. In Experiment 1, participants ranked line patterns (see Figure 6) according to “regularity,” “goodness,” “complexity,” and “pleasantness,” by assigning the numbers 1, 2, and 3 to them (Rank 1 = *most regular, best, least complex, and most pleasant*, respectively). We phrased the question in different ways because the term *goodness* may be unclear to participants. Moreover, these terms correspond to different operationalizations of goodness and are expected to correlate highly (Hamada & Ishihara, 1988; Masame, 1986). We predicted that the average rank would drop as the pattern becomes more degraded because the pattern becomes more complex. In contrast, the holographic approach predicts that the patterns in Figures 6B and 6C should have equally high ranks, and both should be ranked higher than the pattern in Figure 6A. For generality, we also constructed analogous dot patterns. Each individual dot pattern consisted of 10 dots, repeated three times. We then degraded the patterns by moving one or all but one of the dots.

Patterns were presented on separate A4 sheets of paper, with order counterbalanced. Line and dot patterns were presented separately. The order of ranking tasks (goodness, complexity, etc.) was randomized. Participants ranked the patterns by writing 1, 2, or 3 next to each stimulus. Each pattern was judged by at least 23 observers. In the present and all subsequent experiments, the participants were undergraduates at the Universities of Birmingham and Warwick (aged 18–25 years) and were naive to the purpose of the experiments.

Results and discussion. Table 1 shows average rankings of the patterns in Figure 6. Perfect repetitions received top rankings in all tasks. Slightly degraded repetitions were second, and the most degraded patterns were ranked third. The intertask correlations averaged .94 (range, .74–1.00), indicating that our results were not an artifact of the phrasing of question. The only small deviation to this pattern concerns the pleasantness of dot patterns. Here, the perfect repetition and the slightly damaged version received almost equal rankings (1.6 and 1.7, respectively). This may be partially due to the way participants interpreted pleasantness. Some participants may have considered perfect repetitions to be somewhat boring (e.g., see the fine balance between regularity and irregularity in art; Shubnikov & Koptsik, 1974).

The results fit the simplicity view of goodness. As patterns become more complex, their goodness reduces. In contrast, the predictions of the holographic account (van der Helm & Leeuwenberg, 1996, p. 449) are refuted: Participants did not perceive carefully degraded patterns as better than perfect repetitions. These results count directly against the way van der Helm and Leeuwenberg (1996) uncoupled goodness from simplicity.

Claim 3: The Number of Pattern Elements Has No Influence on the Goodness of Bilateral Symmetry and Alternation, Whereas It Has a Major Influence on the Goodness of Repetition (van der Helm & Leeuwenberg, 1996, p. 445)

Bilateral symmetries grow pointwise, so that the number of identities is proportional to the number of pattern elements. Each extra symmetrical pair contributes one identity, and W therefore stays constant at $1/2$. By contrast, repetitions grow blockwise, and each block contributes only one identity, regardless of the number of elements within the block. Therefore, the more elements in a repetition subpattern, the lower W . However, we believe it may not be the total number of elements that is crucial to the goodness of a repetition. Consider Figure 7A, which is 16 elements wide and 4 elements high, and compare it with Figures 7B and 7C, of which the dimensions are 8×8 and 4×16 , respectively. All patterns

Table 1
Mean Rankings for Experiment 1

Goodness measure	Line patterns			Dot patterns		
	A	B	C	A	B	C
Regularity	1.0	2.0	3.0	1.0	2.3	2.7
Goodness	1.5	1.8	2.7	1.5	1.9	2.6
Complexity	1.3	2.0	2.7	1.2	2.0	2.8
Pleasantness	1.5	1.8	2.7	1.6	1.7	2.7

Note. Columns A, B, and C correspond to the manipulations exemplified in Figure 6.

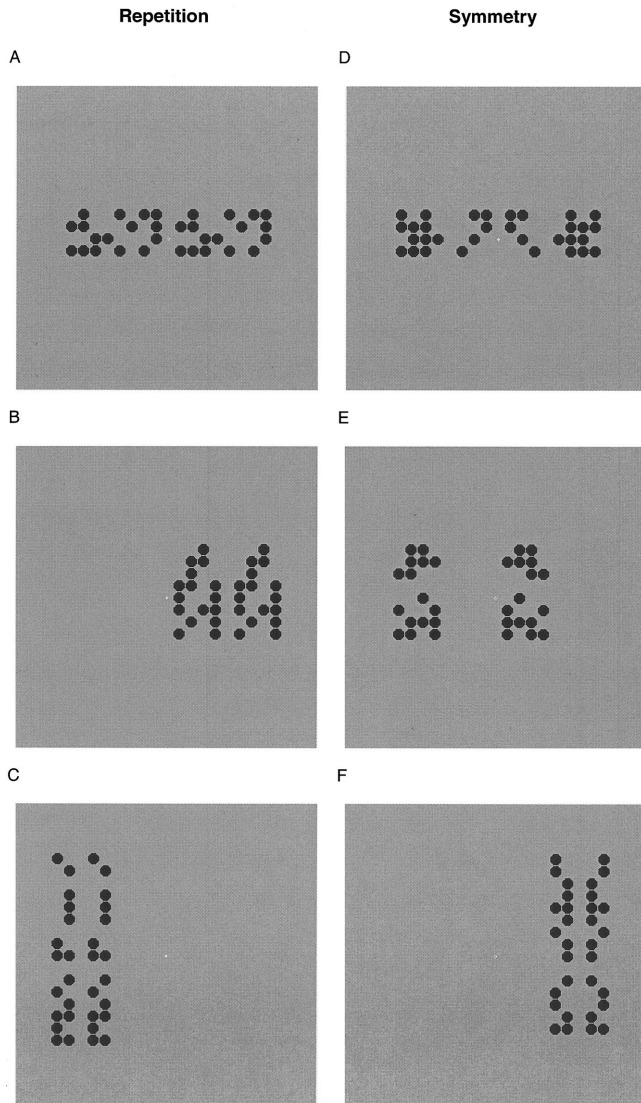


Figure 7. Stimuli examples used in Experiment 2. A shows a repetition that is maximally 16 elements wide and 4 elements high. This pattern would always be presented centrally. B shows a repetition of exactly the same number of elements but now arranged in an 8 × 8 array. In this example, the pattern is presented peripherally, to control for eccentricity effects (eccentricity control condition). C shows a 4 × 16 configuration presented in the periphery. D, E, and F illustrate similar conditions for the symmetry task. Moreover, E illustrates the eccentricity plus distance control manipulation, in which one pattern half was placed at maximum eccentricity and the other half was placed at a distance of half a pattern length.

have the same number of elements and should thus have equal *W*. Yet, their goodness appears to become progressively better. Perhaps more crucial than is that the patterns differ in their 2-D spatial layout. This leads to an important difference between the patterns: namely, the number of elements in between the two members of a to-be-matched pair. In Figure 7A, to match a dot in one pattern with its counterpart in the other, one must skip seven irrelevant intervening dots. In Figure 7B, the number of interfering elements

is three, and in Figure 7C, it is just one. We predict that the number of intervening elements strongly affects the goodness of repetition. This prediction stems from studies of bilateral symmetry perception showing that the area near the axis is important for accurate symmetry detection (e.g., Jenkins, 1982; Julesz, 1971; Wenderoth, 1995; see *Claim 2*). In this area, the to-be-matched dot pairs have few intervening elements, allowing for efficient local processing (see also Wagemans et al., 1993). Similarly, local processing is possible with our narrow but not wide repetitions. If local processing is important, we should see performance vary with pattern width. In contrast, pattern width should have relatively little effect on bilateral symmetries because the local proximity of dots near the axis remains intact (see Figures 7D, 7E, and 7F). This was tested in Experiment 2.

Experiment 2

Method. Eight observers saw patterns such as those in Figure 7 (new patterns were randomly generated for each participant) and decided whether they were regular (symmetrical or repeated) or not (random), by pressing one of two possible response keys. The stimuli were maximally 6.1° wide by 6.1° high and presented for 250 ms, to minimize eye-movement effects. Participants had to distinguish the regular (repeated, mirror-reversed) patterns from irregular (random) patterns and were encouraged to be accurate and fast, with stress on accuracy. One complication is that the wider 16 × 4 stimuli inevitably extend further into the periphery, which might impair detection (e.g., because of decreased acuity or the increased distribution of attention). To control for this, we included an eccentricity control condition, in which the 8 × 8 and 4 × 16 patterns were positioned as far in the periphery as the 16 × 4 pattern (randomly to the left or right of fixation) as well as an eccentricity plus distance control condition, in which the two pattern halves were presented at a distance from each other equal to the 16 × 4 condition (see Figure 7). Note that further eccentricity and attention effects should be controlled for by the symmetry condition, against which the repetition condition was compared. The repetition and symmetry detection tasks were presented in separate blocks in counterbalanced order, with all other trial types randomly mixed within each block. There were 15 trials for each combination of conditions.

Results and discussion. Figure 8 shows percentage error and RTs. There was an eccentricity main effect for detection accuracy, $F(2, 14) = 16.3, MSE = 1.2, p < .001$. Overall, only slightly more errors were made on peripheral patterns than on patterns presented at fixation, with no further interactions. We therefore collapsed the data across the eccentricity conditions. As indicated by Figure 8, more errors were made in the repetition than the symmetry condition, $F(1, 7) = 22.0, MSE = 8.0, p < .01$. This is consistent with earlier findings that repetition is worse than symmetry (e.g., Corballis & Roldan, 1974). There was also a significant effect of pattern width, with more errors for wider patterns, $F(2, 14) = 66.1, MSE = 1.4, p < .001$. This effect was much larger for repetitions than bilateral symmetries, $F(2, 14) = 10.8, MSE = 3.7, p = .001$. In the symmetry condition, errors ranged from 9% for the narrow patterns to 17% for the wide patterns; in the repetition condition, errors rose from 15% to 44%. RTs showed a similar pattern: The rise in RTs was significantly steeper in the repetition condition than in the symmetry condition, $F(2, 14) = 5.3, MSE = 4,952, p = .02$ (which did not interact with eccentricity, $p = .46$).

The results go against the holographic account, which predicts constant performance across constant numbers of pattern elements. In contrast, error rates and RTs rose sharply when the spatial configuration was such that many dots fell in between the to-be-

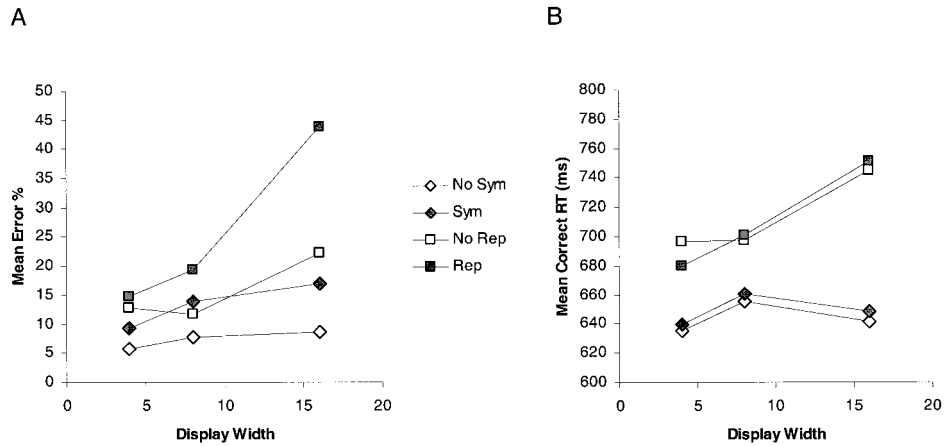


Figure 8. Results of Experiment 2, with error percentage data (A) and reaction time (RT) data (B). No Sym = symmetry condition, irregular patterns; Sym = symmetry condition, regular patterns; No Rep = repetition condition, irregular patterns; Rep = repetition condition, regular patterns.

matched members of a dot pair (the 16×4 configuration). When these members were close to each other (the 8×8 and 4×16 configurations), performance resembled that of the symmetry condition much more.

These results may also explain why Glass patterns are generally so good. A Glass pattern may not be fundamentally different from a normal translation; as long as translation distances are modest, the regularity is easily detectable. Again, the presence or absence of interfering dot pairs seems crucial (Glass, 1969; Maloney et al., 1987). Notice that according to van der Helm and Leeuwenberg (1996), Glass patterns are coded in a fundamentally different way, by alternation, $\langle (G) \rangle \langle (p1)(p2)(p3) \dots (pn) \rangle$, with $p1 \dots pn$ being the positions of each dot pair and G the Glass relationship between the members of each pair. However, Glass patterns can be constructed with a variety of operations, of which translations and rotations are just two. Complex spiral and optic flow patterns can also be detected (e.g., Glass & Perez, 1973). It is unclear how holography would incorporate these without resorting to very complex G s in the alternation. Indeed, as the G operator could be any relationship, there is a danger of collapsing back into the transformational approach, with which van der Helm and Leeuwenberg contrast their account. Moreover, without restrictions on G , we are back where we began, namely, at an unlimited set of coding rules.

Experiment 3

In Experiment 2, we tested (and falsified) holography’s prediction that the goodness of repetition remains constant as long as the number of elements is not altered. In Experiment 3, we test holography’s prediction that bilateral symmetry detection remains constant even if the number of elements is altered. At first sight, this prediction appears to find support in the literature. For instance, both Jenkins (1982) and Wenderoth (1995) found that adding elements outside the region around the axis of symmetry leads to little change in performance. However, this is probably because of the same reason why the area around the axis is so important for symmetry detection (Julesz, 1971), as it allows for

efficient local processing. In contrast, the holographic prediction is based on the overall constancy of the W ratio ($1/2$, regardless of where additional elements are placed). To test between these alternatives, we used the outline polygons illustrated in Figure 9B. The advantage of outline polygons is that the number of elements can be varied without changing the local relationship to the axis (i.e., there is only one region: the outline). However, Figure 9B suggests that, contrary to holography’s prediction, increasing the number of spikes degrades the goodness of the bilateral symmetry.

Method. To test this more thoroughly, we asked 10 participants to decide as quickly and accurately as possible (by one of two possible

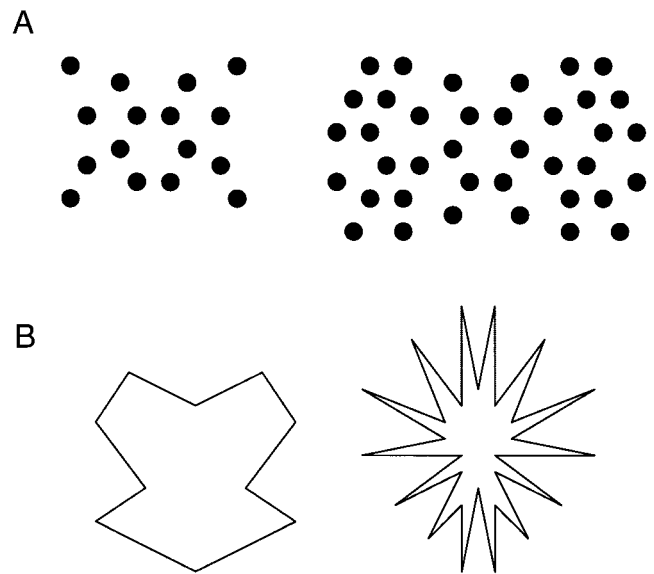


Figure 9. Are extra pattern elements redundant or not? A shows a symmetrical dot pattern (left image). Increasing the number of dots (right image) does not add much to the percept. B shows a polygon (left image). Adding extra elements (right image) appears to affect the overall goodness, contrary to holography’s prediction.

keypresses) whether outline polygons (closely resembling Figure 9B) were perfectly symmetric. Each polygon was presented for 50 ms and was drawn in white on a gray background. We systematically varied two factors: the number of contour elements of the shapes (6, 12, 24, and 48) and the amount of regularity present in the shapes. In the symmetry versus random (SvR) condition, perfect bilateral symmetries were mixed with randomly constructed shapes (of course, within certain constraints). According to holography, perfect symmetries have a goodness of $W = 1/2$, whereas random stimuli have a goodness value of $W = 0$. Thus, the perceptual (goodness) distance between the two types of stimuli would remain constant at the maximum value of $1/2$ across the number of contour elements. We also ran a symmetry versus perturbed (SvP) condition, in which perfect bilateral symmetries were mixed with symmetries in which p spikes were randomly perturbed, with $p = (\text{number of elements})/6$. Thus for 6 contour elements, the perfect symmetry had goodness $W = 3/6$, whereas the perturbed stimulus would receive $W = 2/6$. For 12 contour elements, the values would be $6/12$ and $4/12$, respectively, and so on. In other words, the perceptual distance again remained constant across number of elements, but now at the smaller value of $1/6$. Holography predicts that because the differences in goodness are unaffected by the number of elements and as the overall goodness remains constant, symmetry detection should remain constant. A simplicity-based approach predicts that objects with more elements are more complex and hence that comparisons will be more difficult, in both the SvR and SvP conditions. Here, *elements* may mean more contour elements but also more object parts, such as spikes and concavities.

Results and discussion. Figure 10 shows a steady increase in both RTs and errors with increasing number of contour elements, regardless of condition: SvR, $F(1.6, 15.8) = 47.6$, $MSE = 636.3$, $p < .001$; SvP, $F(1.3, 11.8) = 12.8$, $MSE = 59.5$, $p < .01$. (Where fractionated, degrees of freedom were subject to a Greenhouse–Geisser correction for sphericity violations.) The results are at odds with holography’s prediction of constant performance across number of elements. Instead, these findings are consistent with a simplicity-based account. Adding more contour elements increased shape complexity by increasing the number of elements per shape and by making the shapes more “spiky” or less “compact” (e.g., as measured by the squared perimeter divided by area). Several studies have shown that compactness strongly influences perceived complexity (Attneave, 1957; e.g., Attneave & Arnoult, 1956; see also Hulleman & Boselie, 1999; Wagemans, Lamote, &

Van Gool, 1997). For instance, Hulleman and Boselie (1999) asked participants to choose the most regular of two polygons. Where the regularity was quite difficult to find, participants preferred the most compact of the two shapes, regardless of their regularity.

We conclude from Experiments 2 and 3 that overall pattern goodness is highly dependent on the 2-D spatial layout of the pattern. Because holography deals only with 1-D symbol sequences, it cannot directly account for 2-D effects. Van der Helm and Leeuwenberg (1996) could argue that there are ways to indirectly map 1-D symbol sequences onto 2-D patterns. According to their article (van der Helm & Leeuwenberg, 1996), such a spatial-mapping procedure must follow a spatially contiguous path through the pattern elements (see the General Discussion section). The spatial contiguity may then be different for the different spatial layouts of Figure 7, perhaps even leading to different codes. But, if goodness depends on spatial contiguity, there is a danger that the holography may itself be explanatorily redundant.

Claim 4: Repetitions Benefit More From Extra Local Regularity Than Bilateral Symmetry (van der Helm & Leeuwenberg, 1996, p. 446)

One of holography’s most interesting predictions is that local regularities contribute more to the goodness of repetitions than to the goodness of bilateral symmetries. Consider the bilateral symmetry *abcdef fedcba*, coded as $S[(a)(b)(c)(d)(e)(f)]$ with $W = 6/12$, and add a local regularity to its subpatterns: *abba ef fe abba*. The code becomes $S[S[(a)((b))],(e)(f)]$. The two regularities are combined in a so-called transparent hierarchy, and $W = 8/12$ (an increase of $2/12$). Because symmetry has a point structure, the extra identities in the hierarchically nested subpatterns contribute only once. In contrast, if we start from the repetition *abcdef abcdef*, with $W = 1/12$, the extra local symmetry in *abba ef abba ef* will lead to the transparent hierarchical block code $2 * ((S[(a)(b)])(ef))$. Now, there are two additional identities contributing to each block, leading to a total $W = 5/12$ (an increase of $4/12$). According to van der Helm and Leeuwenberg (1996), holography thus explains why

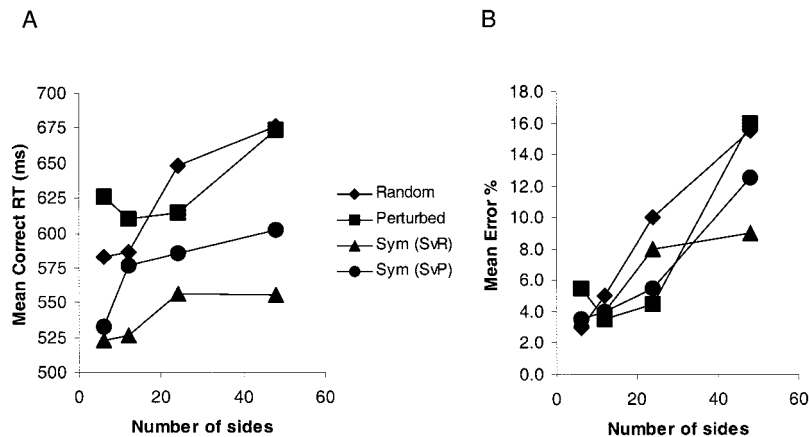


Figure 10. Results of Experiment 3. Mean correct reaction times (RTs; A) and mean error percentage (B) are shown for random patterns, perturbed patterns, and symmetrical (Sym) patterns as compared against random (SvR) or perturbed (SvP) patterns.

adding local symmetries greatly enhances the goodness of repetitions but not bilateral symmetries.

Note, however, that adding local regularities does not change the weight of evidence for the presence of a global repetition per se. For example, the sequence *abba ef cddc pq*, coded as $S[(a)(b)] ef S[(c)(d)] pq$, also has two local symmetries but no global repetition. Its goodness value is $W = 4/12$, which is exactly $1/12$ less than the $W = 5/12$ associated with the improved global repetition example. In other words, the weight of evidence for the repetition itself is still only $1/12$. Thus, holography actually fails to explain why extra local regularities improve repetition detection, and existing data demonstrating that global regularity detection improves with extra regularity can thus be used to falsify the holographic account of goodness (Corballis & Roldan, 1974; Garner & Clement, 1963; Hamada & Ishihara, 1988; Palmer & Hemenway, 1978; Royer, 1966; Wagemans et al., 1991, 1993; also, see Wagemans, 1999, for ways in which the contribution of local regularities may be better explained).

Figure 11 serves as another illustration of the skewed relationship between local regularities and the global goodness value, W , within holography. We trust that most readers will find the symmetry in Figure 11B and see it as much better than Figure 11A. However, according to the holographic approach, Figure 11A is no worse than Figure 11B. This is because Figure 11A actually consists of many local bilateral symmetries in many different orientations (see Figure 11C). Both Figures 11A and 11B therefore receive a goodness value of $W = 1/2$. Note, again, how local regularities do not appear to contribute to the overall goodness.

Claim 5: Transparent Hierarchy Dictates That ISA-Regularities Correspond Directly to ISA-Regularities in the Raw Pattern (van der Helm & Leeuwenberg, 1991, 1996)

We have argued that holography does not predict the increased detectability of hierarchically combined regularities simply because the weight of evidence for one regularity does not provide evidence for the other. Let us again put this problem aside and accept van der Helm and Leeuwenberg's (1996) prediction that hierarchically combined regularities increase the overall goodness

and thus the detectability of the regularities involved. Experiment 4 tests this prediction.

Experiment 4

Consider the Glass pattern in Figure 12A, which consists of randomly placed dot pairs, oriented in the same direction (right diagonal). Such patterns induce a strong perceived regularity and may therefore be assigned high goodness. Van der Helm and Leeuwenberg (1996) encoded Glass patterns using the A-rule (alternation), resulting in the code $\langle(G)\rangle\langle(p1)(p2) \dots (pn)\rangle$. Here, $p1-pn$ indicate positions of one member of each dot pair, and G indicates the Glass relationship between dot pairs (van der Helm & Leeuwenberg, 1996, p. 451). The A-rule has a point structure and therefore leads to a high W within holography. Figures like Figure 12A were used in the Glass condition of Experiment 4.

Figure 12B also consists of dot pairs, but these are now randomly oriented (in any of four directions). These patterns are bilaterally symmetrical about a vertical axis and are referred to as the *sym* condition. In holography, such a pattern is coded via the S-rule, resulting in something like $S[(p1)(p2) \dots (pn)]$, with $p1-pn$ coding the positions of the individual dots or perhaps even the positions plus orientations of complete dot pairs. According to holography, bilateral symmetry has a point structure and should therefore be of high goodness.

Now consider the patterns in Figures 12C–12F. Each pattern consists of a hierarchical combination of a Glass pattern and a bilateral symmetry. For instance, Figure 12C is constructed by taking a horizontal symmetry and translating it slightly in a horizontal direction. Because the Glass pattern and the symmetry have the same orientation, we called this the *aligned* condition. The pattern in Figure 12D is constructed similarly, but the Glass translation (right diagonal) is orthogonal to the axis of symmetry (left diagonal; orthogonal condition). Both patterns can be described by a holographic transparent hierarchy, combining the symmetry and the Glass translation into one code: $S[\langle(G)\rangle/\langle((p1))((p2)) \dots ((pn))\rangle]$. The transparency rule dictates that each regularity described in the code corresponds directly to the same regularity in the pattern. In other words, the regularity must be directly accessible. In fact, because of the transparent hierarchy

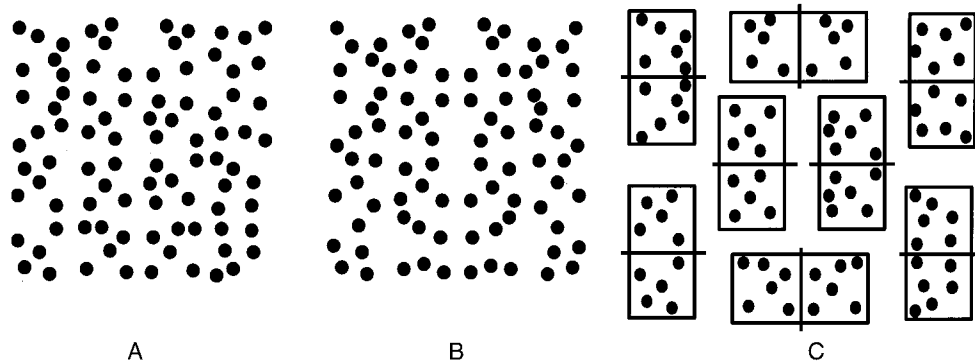


Figure 11. Global versus local symmetry. A shows a pattern consisting of a set of locally symmetrical chunks, which are pointed out in C. B shows a globally symmetrical pattern. The locally and globally symmetrical patterns have the same $W = 1/2$ but differ considerably in figural goodness.

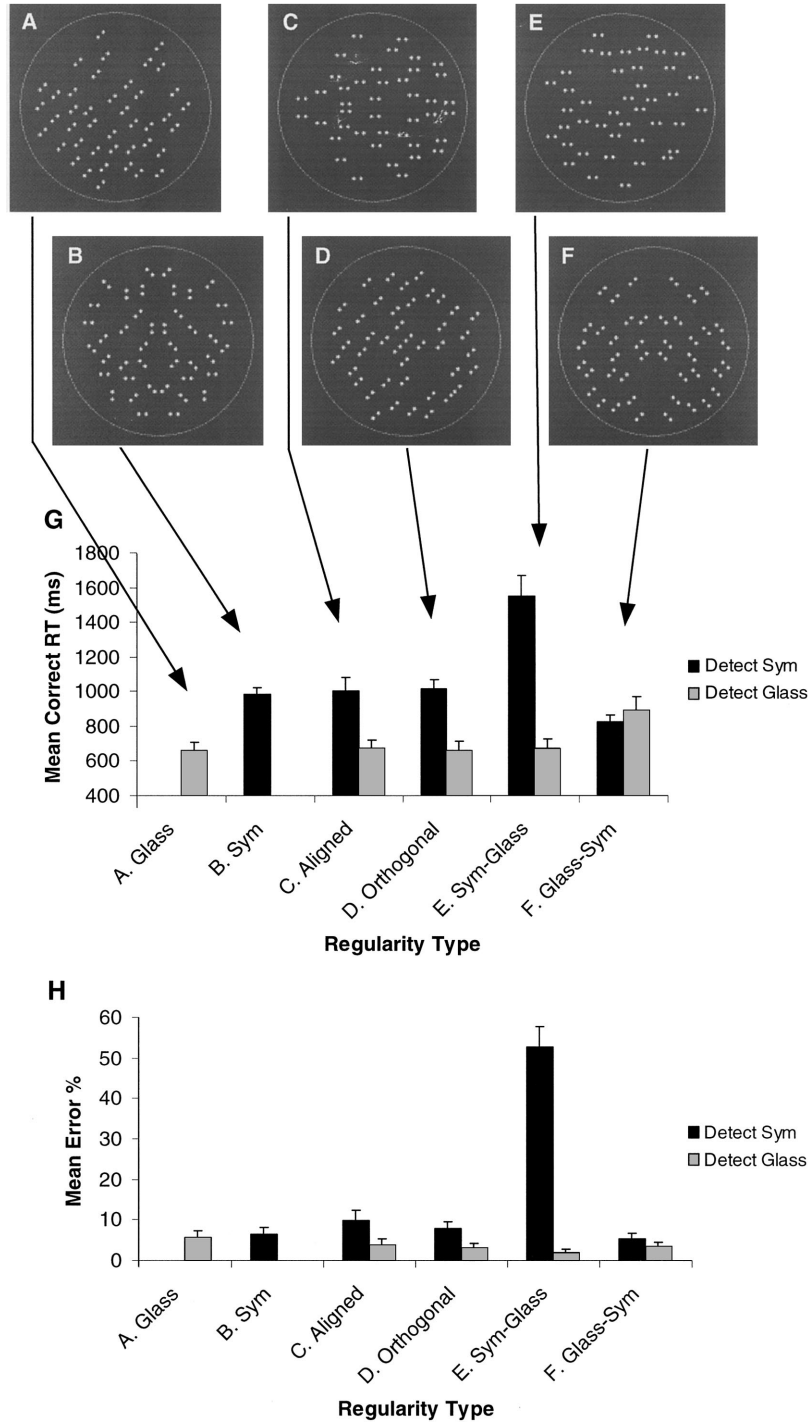


Figure 12. Stimulus examples and results of Experiment 4. A: Baseline Glass pattern. B: Baseline symmetrical pattern. C: Symmetry and Glass are aligned. D: Symmetry orthogonal to Glass pattern. E: Symmetry oblique to Glass pattern. F: Glass pattern mirror reversed. Note that C, D, and E are transparent hierarchies of a symmetry and a Glass pattern. F is also a hierarchy of a symmetry and a Glass pattern, but this hierarchy is not transparent. G: Reaction time (RT) data for each regularity type, for trials on which the target regularity was present. H: Error percentage data for each regularity type, for trials on which the target regularity was present. Error bars indicate one standard error from the mean across participants. In the Detect Sym condition, participants first had to detect the presence of a bilateral symmetry and then point out the direction of the axis. In the Detect Glass condition, participants first had to detect the presence of a Glass pattern and then point out its direction of flow. All types of patterns were included in all conditions.

and the point structure of bilateral symmetry as well as Glass patterns, the code can be turned inside out without affecting the hierarchy: $\langle(G)\rangle/\langle S[(p1)(p2) \dots (pn)]\rangle$. Looking at the patterns in Figures 12C and 12D, one can see that this seems indeed intuitively to be the case, as both regularities seem readily perceivable. According to holography, the transparent hierarchy increases W , from $W = 1/2$ for the patterns in Figures 12A and 12B to $W = 3/4$ for the patterns in Figures 12C and 12D. This improved goodness should thus lead to an improved detection of either regularity, just as additional local symmetries improve the detection of a global symmetry or translation (van der Helm & Leeuwenberg, 1996, pp. 446–454).

Like the patterns in Figures 12C and 12D, the pattern in Figure 12E is a transparent hierarchy of a bilateral symmetry and a Glass pattern. The symmetry (left diagonal) is now translated in a direction oblique to its axis of symmetry (namely horizontally; we call this the *sym-Glass* condition). Holography predicts no difference between the patterns in Figures 12C, 12D, and 12E. This is because 1-D symbol sequences cannot distinguish between different directions of translation. But even if they could, under the rule of transparency, the underlying symmetry should still be readily perceivable for all patterns. In contrast, we predicted that the Glass translation in the pattern in Figure 12E would severely damage the symmetry percept, as it destroys the local symmetry relationships between pairs of dots (cf. the correlational quadrangles proposed by Wagemans et al., 1993). Only by destroying the locally oriented dot pairs (and thus by destroying the Glass regularity) can the symmetry improve, for instance by applying a low-pass filter (for readers who find it hard to see any symmetry, blurring the picture by looking through the eyelashes may help).

Finally, consider the pattern in Figure 12F, which is again a combination of a Glass pattern and a bilateral symmetry. It was constructed by plotting half a Glass pattern and then flipping it across the axis of symmetry (hence, we call it the *Glass-sym* condition). Holography cannot capture this hierarchy because the Glass translations in the pattern halves are in orthogonal directions. Therefore, for instance, holography would be able to encode the Glass pattern in the left half of the display as $\langle(G1)\rangle/\langle(p1)(p2) \dots (pn)\rangle$ and the right half as $\langle(G2)\rangle/\langle(p1)(p2) \dots (pn)\rangle$ (with $G1$ and $G2$ denoting different Glass relationships), but the different Glass directions then block the possibility of coding the symmetry between the halves (i.e., the S-rule cannot cope with $G1$ and $G2$). The overall goodness of this combination would still be quite high but no higher than for the baseline patterns in Figures 12A or 12B (approximately $1/2$). Alternatively, a holographic perceptual system could choose to code the pattern in Figure 12F as a bilateral symmetry instead— $S[(p1)(p2) \dots (pn)]$ —but this precludes any overall coding of the Glass pattern. Therefore, under holography, symmetry detection for the pattern in Figure 12F is not predicted to be any better than for the pattern in Figure 12A.

Method. Nine observers detected, as quickly as they could, the regularities in patterns like those in Figure 12 (see also Locher & Wagemans, 1993, and Wagemans et al., 1993, for similar manipulations). The patterns were circular (4.2° radius) and consisted of white dots on a gray background. Each pattern was randomly generated on each trial and had one of four orientations: vertical, horizontal, left diagonal (45° counterclockwise), and right diagonal (45° clockwise). Participants pressed the space bar when they detected the regularity (this was the point at which we measured RT). The pattern then disappeared, and participants saw a display with five

response options, corresponding to the four orientations plus a “no orientation” option. Participants made an unspeeded choice by pressing one of five keys on the keyboard. The display provided feedback on whether their choice was correct. Each participant performed two tasks on the same set of stimuli (in counterbalanced blocks): a Glass-detection task and a symmetry-detection task. For instance, in the Glass-detection task, when faced with a pattern like that in Figure 12A, participants would press the space bar and then choose “right diagonal.” In the symmetry-detection task, for the pattern in Figure 12B, the correct response would be “vertical.”

Results and discussion. Mean RTs and mean error percentages are plotted in the graphs of the bottom half of Figure 12. The graphs show the data for the conditions in which the to-be-detected regularity was present. The results for the regularity absent conditions were as follows. When participants had to detect a symmetry but only a Glass regularity was present (symmetry absent, see Figure 12A), the RT was 2,247 ms and the error rate was 59%. When participants had to detect a Glass regularity but only a symmetry was present (Glass absent, see Figure 12B), the RT was 1,276 ms and the error rate was 26%. These were mostly time-out errors, as observers tended to keep on looking for a possible presence of the regularity (which also explains the large RTs). Error rates and RTs correlated strongly ($r = .94$), and we therefore limit our report to RTs. There was a significant effect of orientation in the symmetry detection, $F(2.22, 17.7) = 3.9$, $MSE = 61,656$, $p < .05$, but not in Glass detection, $F(2.40, 19.2) = 1.6$, *ns*. Consistent with earlier findings (Palmer & Hemenway, 1978; Royer, 1981; Wagemans, Van Gool, & d’Ydewalle, 1992), vertical symmetries were detected fastest, followed by horizontal and diagonal symmetries. However, orientation did not interact with any other factors, and the analyses reported below were performed on data collapsed across orientation (resulting in at least 40 data points per cell).

First, responses were faster for Glass patterns than for bilateral symmetries, $F(1, 8) = 16.5$, $MSE = 177,958$, $p < .01$. In Figure 12, Glass patterns seem to result in stronger percept than bilateral symmetries. Indeed, they are so strong that they can either destroy (see Figure 12E) or boost a symmetry (see Figure 12F). In contrast, none of the conditions (except *Glass-sym*) suggest that the presence of a bilateral symmetry affects Glass detection, indicating that the relationship between Glass and symmetry is asymmetrical rather than reciprocal. This is inconsistent with holography, which predicts that Glass and bilateral symmetry regularities are equally good and can be mutually exchanged because of transparent hierarchy. Possibly, however, this asymmetry was caused by stimulus parameters, such as dot spacing. Although all patterns were made up of the same randomly placed dot pairs, the average distance between members of a Glass dot pair is shorter than the average distance between dots across the axis of symmetry because of the very nature of these regularities. Holography might be defended by stating that Glass patterns and bilateral symmetries are of equal goodness, other things being equal. But, given the inherently different spatial structure of the two types of regularity, it is difficult to see how other things could ever be equal; hence, this defense risks leading holography into unfalsifiability.

Another possibility is that Glass patterns are perceived in an earlier stage of the visual system. Glass patterns have been associated with optic flow and/or texture segmentation processes,

which may be preattentive and automatic (Dakin, 1997; Glass & Perez, 1973; Julesz, 1981; Prazdny, 1984). In contrast, recent evidence suggests that bilateral symmetry detection involves higher visual processing, needing limited attentional resources (Olivers & van der Helm, 1998). Holography does not account for attentional effects on regularity perception and is thus difficult to test.

We also found no improvements for the hierarchically combined regularities of the aligned, orthogonal, and sym-Glass patterns (see Figures 12C, 12D, and 12E), although their holographic goodness was 1.5 times higher than the sym baseline (see Figure 12B). Whereas performance remained constant in the Glass-detection task (as confirmed by individual t tests comparing it with performance in the standard Glass condition, see Figure 12A, $p > .4$), symmetry detection was much worse for sym-Glass patterns (see Figure 12E), $F(2.2, 17.8) = 38.2$, $MSE = 13,180$, $p < .001$: t test comparing it with the standard sym condition (see Figure 12B), $t(8) = 5.78$, $p < .001$. Both RTs and errors rose sharply. Apparently, it is particularly hard to detect an underlying bilateral symmetry when a Glass regularity on a different hierarchical level destroys the superficial pattern symmetry. This result directly contradicts the holographic prediction that all regularities in a transparent hierarchy are readily available for perception. In the aligned and orthogonal conditions (see Figures 12C and 12D), the overall symmetry was not destroyed by the Glass regularity. Yet, nor did the Glass regularity contribute to better performance for these patterns. This finding too cannot be explained by holography, which predicts improved performance as regularities are combined.

We further found that in the Glass-sym condition (see Figure 12F), symmetry detection did improve, $t(8) = 4.43$, $p < .01$. In this condition, the presence of the Glass pattern aided the perception of symmetry, again contradicting holography's prediction. Because holography cannot combine the symmetry and Glass regularities in this pattern, performance should have been no better than in the standard sym condition (see Figure 12B). Instead, the improvement confirms earlier findings by Locher and Wagemans (1993; see also Wagemans et al., 1993). They presented participants with bilateral symmetries of which the local elements could be oriented randomly or parallel, perpendicular, and oblique to the axis of symmetry (similar to the sym, aligned, orthogonal, and Glass-sym conditions here). Consistent with our results, Locher and Wagemans found an improvement for patterns with oblique elements but not for patterns with parallel and perpendicular elements. They suggested that local elements group on the basis of similarity. In the parallel and perpendicular patterns, such grouping will be strong but independent of the bilateral symmetry. In oblique patterns, however, the different textures will lead to a strong segmentation at the axis of symmetry, thus aiding symmetry detection.

Finally, for the Glass-detection task, performance deteriorated in the Glass-sym condition (see Figure 12F), $t(8) = 4.50$, $p < .01$. This result is analogous to that for the sym-Glass condition (see Figure 12E) in the symmetry-detection task. There, the symmetry was broken up by an incompatible Glass pattern, whereas here the Glass pattern is broken up by an incompatible symmetry (as opposed to the aligned and orthogonal conditions, in which Glass and symmetry are compatible). Performance in the Glass-sym condition may also have been slowed by task demands. As can be

seen in the pattern in Figure 12F, the Glass regularity contains two directions. Although we told participants that they could pick either direction for their response, the ambiguity may have slowed them down.

Claim 6: Weight of Evidence Explains the Symmetry Effect (van der Helm & Leeuwenberg, 1996, p. 450)

Van der Helm and Leeuwenberg (1996) also claimed that holography explains the biases in symmetry perception found by Freyd and Tversky (1984; also Carmody, Nodine, & Locher, 1977; King, Meyer, Tangney, & Biederman, 1976). Freyd and Tversky presented participants with a variety of polygonlike contour shapes, which were bilaterally symmetric except for some perturbations. Participants then matched this reference shape to two target shapes, which were identical to the reference shape save a few small alterations: The more symmetrical target was less perturbed than the reference shape; the less symmetrical target was more perturbed. Participants indicated which target shape best matched the reference. Freyd and Tversky found that preferences changed with the symmetry of the reference stimulus. When the reference was highly symmetrical, observers preferred the even more symmetrical target. When the reference was less symmetrical, observers preferred the even less symmetrical option. Freyd and Tversky concluded that there is a symmetry bias for highly symmetrical stimuli and an asymmetry bias for less symmetrical stimuli.

Van der Helm and Leeuwenberg (1996, see also 1999) argued that W predicts these effects and moreover provides ecological support for holography. They assigned a standard goodness value to Freyd and Tversky's (1984) reference shape: $W_{\text{ref}} = E/n$, with E being the number of identities and n the number of pattern elements (including some noise elements). Subsequently, the more and less symmetrical target stimuli are assigned $W_{\text{more}} = (E + x)/(n + x)$ and $W_{\text{less}} = (E - x)/(n - x)$, respectively, with x being an added or subtracted element (x is much smaller than n). With E , n , and x all positive, algebra shows that $W_{\text{more}} - W_{\text{ref}} < W_{\text{ref}} - W_{\text{less}}$. Thus, if dissimilarity is measured by difference in W between patterns, the reference is most similar to the more symmetrical stimulus: a symmetry bias.

However, notice that this analysis is neutral concerning the degree of symmetry of the reference shape, which Freyd and Tversky (1984) found to be critical. Hence, highly asymmetrical reference shapes are also predicted (incorrectly) to lead to a symmetry bias. To account for this, van der Helm and Leeuwenberg (1996) proposed that Freyd and Tversky must have constructed their less symmetrical stimuli in a fundamentally different way, so that W_{more} and W_{less} should also be derived differently. They altered W purely by adding or removing noise elements, while the number of identities remains constant. Thus, $W_{\text{more}} = E/(n - x)$, whereas $W_{\text{less}} = E/(n + x)$. This reverses the bias because $W_{\text{more}} - W_{\text{ref}} > W_{\text{ref}} - W_{\text{less}}$.

There are many other methods of calculating W , some leading to symmetry biases and some leading to asymmetry biases. The point is that without independent justification for switching the method of calculation, holography does not explain the empirical data but shows that it is general enough to predict any pattern, at will.

A second problem is that the difference in W between patterns seems a poor measure of similarity: For example, a Glass pattern

and a bilaterally symmetrical pattern may both have $W = 1/2$ yet be judged very dissimilar.

Nevertheless, van der Helm and Leeuwenberg (1996, 1999) relied on this insubstantial foundation to provide ecological support for holography. But, even were this foundation solid, their subsequent ecological argument presents difficulties. They suggested that in the real world the symmetry bias may help a predator spot its partially hiding prey by enhancing the symmetry and hence the “objectness” of the stimulus (van der Helm & Leeuwenberg, 1996, p. 452). But they also suggested that, at the same time, the asymmetry bias has survival value, as it decreases the symmetry (and therefore “objectness”) of the prey, so that the predator is less likely to perceive it (van der Helm & Leeuwenberg, 1996, p. 452). These suggestions are clearly incompatible. Moreover, whether a symmetry or asymmetry bias is observed depends, in Tversky and Freyd’s (1984) laboratory study, on the degree of symmetry of the given reference stimulus. However, van der Helm and Leeuwenberg (1996) provided no link to which reference stimulus is appropriate in natural contexts.

General Discussion

We have argued that despite its strong mathematical basis and broad empirical scope, the holographic approach to figural goodness suffers from a range of fundamental theoretical and empirical problems. The key explanatory principles, such as the distinction between point and block structures, do not follow from the theory. The theory shows internal inconsistencies in relation to noise resistance and hierarchically combined regularities. Finally, across the range of empirical phenomena that the theory addresses, the account either is flexible enough to capture any pattern of data or is empirically falsified.

We believe that most of these problems arise because van der Helm and Leeuwenberg’s (1996) “theory does not prescribe in detail how raw 1-D symbolic representations are to be obtained [from 2-D patterns]” (p. 443), despite their earlier claims that its 1-D “principles . . . can be generalized straightforwardly to 2-D pattern regularity” (p. 429). This generalization has yet to be demonstrated. Currently, the account places no real constraint on the mapping between 2-D stimuli and 1-D symbol sequences, thus breaking the link between the mathematics of the theory and psychological predictions. This point has been noted by Wagemans (1999), who pointed out that holography can detect only certain regularities if the symbols are placed in the right order. We have shown that the symbol order is actually rather arbitrary, as are associated block or point structures.

Nevertheless, van der Helm and Leeuwenberg (1996, 1999) have made initial attempts to describe a spatial-mapping procedure, but unlike transparency and holography, it has (as yet) no formal definition. Van der Helm and Leeuwenberg (1996) related it to proximity but, more generally, also to “connectedness” (p. 443; 1999, p. 625). For instance, van der Helm and Leeuwenberg (1996) defined a dot pattern as spatially contiguous if the dots can be connected by a path, without the path ever crossing itself and without it ever visiting one dot more than once. Further restrictions are that the path should stay within one half of the pattern first, for bilateral symmetries as well as repetitions, before moving to the other half. Especially this latter restriction seems crucial in the detection of the different regularities and the occurrence of point

versus block structures (cf. Wagemans, 1999). Yet, Experiment 2 has already demonstrated that this cannot be the whole story. In that experiment, we presented repetitions differing only in width to height ratio, in which the total number of dots remained constant but the number of dots between two repeated dots varied. All patterns could be holographically coded in the same way—following van der Helm and Leeuwenberg’s (1996) spatially contiguous path first through one half and then through the other—and should thus have led to the same goodness. Yet, the goodness of the patterns differed substantially. The fewer interfering elements, the better detection became. Explaining this requires reference to the 2-D structure of the patterns. But, spatial contiguity, as described by van der Helm and Leeuwenberg (1996), does not distinguish between one and two dimensions.

The contour elements of outline shapes do possess an intrinsic connectedness, and perhaps a spatially contiguous symbol sequence may be more easily constructed by following such contours. However, Hulleman and Boselie (1999) have recently demonstrated that regularities that are restricted to the contour of a pattern do not always determine regularity detection. For instance, equilateral polygons (but with random angles) should result in a high goodness value, as they can be coded through the A-rule. Contrary to this prediction, Hulleman and Boselie (1999) found that they were indistinguishable from random polygons. Hulleman and Boselie further found that shapes with a high degree of bilateral symmetry along the contour but that do not have corresponding object parts (as defined by convexities and concavities in the contour; see also Baylis & Driver, 1995) are also no better than random polygons. Moreover, we have demonstrated in Figure 3 that even with outline figures there can be many different spatially contiguous paths (e.g., U, N, and Z routes).

Another defense of holography might attempt to connect a holographic theory of *representation* to a theory of visual *processing*. This line is adopted by van der Helm and Leeuwenberg (1999) in response to Wagemans (1999). Van der Helm and Leeuwenberg proposed a synthesis between Wagemans’s bootstrap model of symmetry detection (e.g., Wagemans, 1995; Wagemans et al., 1993) and the holographic theory of regularity detection, arguing that certain bootstrapping procedures could be adapted to lead to point and block structures as in the holographic framework. Van der Helm and Leeuwenberg (1999) claimed that such a synthesis would solve many of holography’s processing problems and provide a formal justification for the bootstrap model. According to the resulting holographic-bootstrapping model, the visual system searches for correlational quadrangles between pattern elements. This search starts with a chunk of the symbol sequence and propagates from there. According to van der Helm and Leeuwenberg (1999), the propagation of quadrangles proceeds exponentially in bilateral symmetries and alternations, whereas it proceeds linearly in repetitions.

However, in the light of the analysis developed here, this account does not seem compelling. First, there is no a priori reason for why repetitions should only propagate linearly: Glass patterns, which have the same quadrangles as repetitions, do not propagate in this way. The reason van der Helm and Leeuwenberg (1999) offered is that the bootstrapping propagation should be “in agreement with the way representations grow holographically” (p. 627). In other words, bootstrapping should follow a block structure for repetitions and a point structure for Glass patterns and symmetries.

Leaving aside concerns of circularity here, note that there is no single holographic way in which specific regularities grow, as discussed in the present article. Moreover, the difference in propagation speed between Glass patterns and repetitions can be more easily, and certainly more elegantly, explained by the difference in translation distance (both spatially and temporally; e.g., Jenkins, 1983a, 1983b). Bootstrapping is slower for long-distance repetitions because the members of the virtual quadrangles are harder to find when there are more interfering elements in between. Bilateral symmetries, conversely, are quite easy to find because bootstrapping can start locally, without interference (Wagemans et al., 1993). The visual system solves many global problems by tackling them locally. This works well for bilateral symmetries and Glass patterns but is problematic for repetitions. Of course, *local* may be seen at different scales (Dakin, 1997; Dakin & Watt, 1994).

Perhaps one day the problem of spatial mapping will be solved within holography. If so, however, we suggest that such a consistent spatial-mapping procedure may itself be a criterion for regularity, which is likely to bypass the need for any holographic criteria. In other words, we suspect that it will be difficult to augment holography successfully without making it redundant. In view of van der Helm and Leeuwenberg's (1996) representational stance, it may be more promising to return to the traditional assumption that figural goodness is a function of simplicity and to use processing assumptions to explain away apparent goodness advantages of symmetry over the equally simple repetition. Alternatively, it may be that a purely process-oriented (Wagemans, 1995) or transformational approach (Palmer, 1983) is ultimately more fruitful. In our view, the visual system represents the simplest regularity it can find. Finding a regularity, however, is heavily determined by processing factors involved in attention, spatial proximity, scale, optic flow, orientation, perspective, continuity, common fate, and object-part correspondence (Baylis & Driver, 1995; Dakin & Watt, 1994; Glass & Perez, 1973; Olivers & van der Helm, 1998; Wagemans et al., 1992, 1993; Wenderoth, 1994). It is important to note that if a regularity is easily found, its goodness will be high. If a regularity is difficult to find, it will not be represented at all, and its goodness will be low.

References

- Attneave, F. (1954). Some informational aspects of visual perception. *Psychological Review*, *61*, 183–193.
- Attneave, F. (1955). Symmetry, information, and memory for patterns. *American Journal of Psychology*, *68*, 209–222.
- Attneave, F. (1957). Physical determinants of the judged complexity of shapes. *Journal of Experimental Psychology*, *53*, 221–227.
- Attneave, F., & Arnoult, M. D. (1956). The quantitative study of shape and pattern perception. *Psychological Bulletin*, *53*, 452–471.
- Barlow, H. B., & Reeves, B. C. (1979). The versatility and absolute efficiency of detecting mirror symmetry in random dot displays. *Vision Research*, *19*, 783–793.
- Baylis, G. C., & Driver, J. (1994). Parallel computation of symmetry but not repetition within single visual shapes. *Visual Cognition*, *1*, 377–400.
- Baylis, G. C., & Driver, J. (1995). Obligatory edge assignment in vision: The role of figure and part segmentation in symmetry detection. *Journal of Experimental Psychology: Human Perception and Performance*, *21*, 1323–1342.
- Bruce, V. G., & Morgan, M. J. (1975). Violations of symmetry and repetition in visual patterns. *Perception*, *4*, 239–249.
- Carmody, D. P., Nodine, C. F., & Locher, P. J. (1977). Global detection of symmetry. *Perceptual and Motor Skills*, *45*, 1267–1273.
- Chater, N. (1996). Reconciling simplicity and likelihood principles in perceptual organization. *Psychological Review*, *103*, 566–581.
- Chater, N. (1999). The search for simplicity: A fundamental cognitive principle? *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, *52(A)*, 273–302.
- Chipman, S. (1977). Complexity and structure in visual patterns. *Journal of Experimental Psychology: General*, *106*, 269–301.
- Chipman, S. F., & Mendelson, M. J. (1979). Influence of six types of visual structure on complexity judgments in children and adults. *Journal of Experimental Psychology: Human Perception and Performance*, *5*, 365–378.
- Corballis, M. C., & Roldan, C. E. (1974). On the perception of symmetrical and repeated patterns. *Perception & Psychophysics*, *16*, 136–142.
- Dakin, S. C. (1997). The detection of structure in glass patterns: Psychophysics and computational models. *Vision Research*, *37*, 2227–2246.
- Dakin, S. C., & Watt, R. J. (1994). Detection of bilateral symmetry using spatial filters. *Spatial Vision*, *8*, 393–413.
- Freyd, J., & Tversky, B. (1984). Force of symmetry in form perception. *American Journal of Psychology*, *97*, 109–126.
- Garner, W. R. (1974). *The processing of information and structure*. Potomac, MD: Erlbaum.
- Garner, W. R., & Clement, D. E. (1963). Goodness of pattern and pattern uncertainty. *Journal of Verbal Learning and Verbal Behavior*, *2*, 446–452.
- Glass, L. (1969). Moire effect from random dots. *Nature*, *223*, 578–580.
- Glass, L., & Perez, L. (1973). Perception of random dot interference patterns. *Nature*, *246*, 360–362.
- Glass, L., & Switkes, E. (1976). Pattern recognition in humans: Correlations which cannot be perceived. *Perception*, *5*, 67–72.
- Hamada, J., & Ishihara, T. (1988). Complexity and goodness of dot patterns varying in symmetry. *Psychological Research*, *50*, 155–161.
- Hardonk, M. (1999). *Cross-cultural universals of aesthetic appreciation in decorative band patterns*. Unpublished doctoral dissertation, University of Nijmegen, Nijmegen, the Netherlands.
- Hatfield, G. C., & Epstein, W. (1985). The status of the minimum principle in the theoretical analysis of visual perception. *Psychological Bulletin*, *97*, 155–186.
- Hulleman, J., & Boselie, F. (1999). Perceived shape regularity does not depend on regularities along the contour. *Perception*, *28*, 711–724.
- Jenkins, B. (1982). Redundancy in the perception of bilateral symmetry in dot textures. *Perception & Psychophysics*, *32*, 171–177.
- Jenkins, B. (1983a). Spatial limits to the detection of transpositional symmetry in dynamic dot textures. *Journal of Experimental Psychology: Human Perception and Performance*, *9*, 258–269.
- Jenkins, B. (1983b). Temporal limits to the detection of correlation in transpositionally symmetric dynamic dot textures. *Perception & Psychophysics*, *33*, 79–84.
- Julesz, B. (1971). *Foundations of cyclopean perception*. Chicago: Chicago University Press.
- Julesz, B. (1981). A theory of preattentive texture discrimination based on first-order statistics of textons. *Biological Cybernetics*, *41*, 131–138.
- Kahn, J. I., & Foster, D. H. (1986). Horizontal-vertical structure in the visual comparison of rigidly transformed patterns. *Journal of Experimental Psychology: Human Perception and Performance*, *12*, 422–433.
- King, M., Meyer, G. E., Tangney, J., & Biederman, I. (1976). Shape constancy and a perceptual bias towards symmetry. *Perception & Psychophysics*, *19*, 129–136.
- Koffka, K. (1963). *Principles of Gestalt psychology* (5th ed.). London: Routledge and Kegan Paul. (Original work published 1935)
- Leeuwenberg, E. (1969). Quantitative specification of information in sequential patterns. *Psychological Review*, *76*, 216–220.

- Leeuwenberg, E., & Boselie, F. (1988). Against the likelihood principle in visual form perception. *Psychological Review*, *95*, 485–491.
- Leeuwenberg, E., & Buffart, H. (1983). An outline of coding theory. In H. G. Geissler & H. Buffart (Eds.), *Modern issues in perception* (pp. 25–47). Amsterdam: North-Holland.
- Locher, P. J., & Wagemans, J. (1993). Effects of element type and spatial grouping on symmetry detection. *Perception*, *22*, 565–587.
- Mach, E. (1897). Contributions to the analysis of sensations (1st ed.; C. M. Williams, Trans.). Chicago: Open Court. (Original work published 1886)
- Maloney, R. K., Mitchison, G. J., & Barlow, H. B. (1987). Limit to the detection of Glass patterns in the presence of noise. *Journal of the Optical Society of America*, *4*, 2336–2341.
- Masame, K. (1986). Rating of symmetry as continuum. *Tohoku Psychologica Folia*, *45*, 17–27.
- Olivers, C. N. L., & van der Helm, P. A. (1998). Symmetry and selective attention: A dissociation between effortless perception and serial search. *Perception & Psychophysics*, *60*, 1101–1116.
- Palmer, S. E. (1982). Symmetry, transformation, and the structure of perceptual systems. In J. Beck (Ed.), *Organization and representation in perception* (pp. 95–144). Hillsdale, NJ: Erlbaum.
- Palmer, S. E. (1983). The psychology of perceptual organization: A transformational approach. In J. Beck, B. Hope, & A. Rosenfeld (Eds.), *Human and machine vision* (pp. 269–339). New York: Academic Press.
- Palmer, S. E. (1991). Goodness, Gestalt, groups, and Garner: Local symmetry subgroups as a theory of figural goodness. In G. R. Lockhead & J. R. Pomerantz (Eds.), *The perception of structure* (pp. 23–39). Washington, DC: American Psychological Association.
- Palmer, S. E., & Hemenway, K. (1978). Orientation and symmetry: Effects of multiple, rotational, and near symmetries. *Journal of Experimental Psychology: Human Perception and Performance*, *4*, 691–702.
- Pomerantz, J. R. (1977). Pattern goodness and speed of encoding. *Memory & Cognition*, *5*, 235–241.
- Pomerantz, J. R., & Kubovy, M. (1986). Theoretical approaches to perceptual organization. In K. R. Boff, L. Kaufman, & J. P. Thomas (Eds.), *Handbook of perception and human performance* (pp. 36.1–36.46). New York: Wiley.
- Prazdny, K. (1984). On the perception of Glass patterns. *Perception*, *13*, 469–478.
- Restle, F. (1979). Coding theory of the perception of motion configurations. *Psychological Review*, *86*, 1–24.
- Royer, F. L. (1966). Figural goodness and internal structure in perceptual discrimination. *Perception & Psychophysics*, *1*, 311–314.
- Royer, F. L. (1981). Detection of symmetry. *Journal of Experimental Psychology: Human Perception and Performance*, *7*, 1186–1210.
- Shubnikov, A. V., & Koptsik, V. A. (1974). *Symmetry in science and art*. New York: Plenum Press.
- Simon, H. A. (1972). Complexity and the representation of patterned sequences of symbols. *Psychological Review*, *79*, 369–382.
- Tyler, C. W., Hardage, L., & Miller, R. T. (1995). Multiple mechanisms for the detection of mirror symmetry. *Spatial Vision*, *9*, 79–100.
- van der Helm, P. A. (1988). *Accessibility and simplicity of visual structures*. Unpublished doctoral dissertation, University of Nijmegen, Nijmegen, the Netherlands.
- van der Helm, P. A., & Leeuwenberg, E. L. J. (1991). Accessibility: A criterion for regularity and hierarchy in visual pattern codes. *Journal of Mathematical Psychology*, *35*, 151–213.
- van der Helm, P. A., & Leeuwenberg, E. L. J. (1996). Goodness of visual regularities: A nontransformational approach. *Psychological Review*, *103*, 429–456.
- van der Helm, P. A., & Leeuwenberg, E. L. J. (1999). A better approach to goodness: Reply to Wagemans (1999). *Psychological Review*, *106*, 622–630.
- van der Helm, P. A., van Lier, R. J., & Leeuwenberg, E. L. J. (1992). Serial pattern complexity: Irregularity and hierarchy. *Perception*, *21*, 517–544.
- Wagemans, J. (1995). Detection of visual symmetries. *Spatial Vision*, *9*, 9–32.
- Wagemans, J. (1999). Toward a better approach to goodness: Comments on van der Helm and Leeuwenberg (1996). *Psychological Review*, *106*, 610–621.
- Wagemans, J., Lamote, C., & Van Gool, L. (1997). Shape equivalence under perspective and projective transformations. *Psychonomic Bulletin & Review*, *4*, 248–253.
- Wagemans, J., Van Gool, L., & d'Ydewalle, G. (1991). Detection of symmetry in tachistoscopically presented dot patterns: Effects of multiple axes and skewing. *Perception & Psychophysics*, *50*, 413–427.
- Wagemans, J., Van Gool, L., & d'Ydewalle, G. (1992). Orientational effects and component processes in symmetry detection. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, *44(A)*, 475–508.
- Wagemans, J., Van Gool, L., Swinnen, V., & Van Horebeek, J. (1993). Higher-order structure in regularity detection. *Vision Research*, *33*, 1067–1088.
- Wenderoth, P. (1994). The salience of vertical symmetry. *Perception*, *23*, 221–236.
- Wenderoth, P. (1995). The role of pattern outline in bilateral symmetry detection with briefly flashed dot patterns. *Spatial Vision*, *9*, 57–77.
- Wolfe, J. M., & Friedman-Hill, S. R. (1992). On the role of symmetry in visual search. *Psychological Science*, *3*, 194–198.

Received June 2, 2000

Revision received March 19, 2001

Accepted July 31, 2001 ■